REV-01
MSM/22/27

2024/12

SET A

M.Sc. MATHEMATICS
FIRST SEMESTER
LINEAR ALGEBRA
MSM – 102

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. For a linear transformation $T: R^{10} \rightarrow R^{6}$, the kernel is having dimension 5. Then the dimension of the range of T is
   a. 5
   b. 6
   c. 4
   d. 2

2. Dimension of vector subspace $W = \left\{(a, b, c) : a = b = c\right\}$ of vector space $R^{3}(R)$ equals
   a. 0
   b. 1
   c. 2
   d. 3

3. The singleton set $\{\alpha\}$ is linearly dependent iff
   a. $\alpha = 0$
   b. $\alpha \neq 0$
   c. α is scalar
   d. None

4. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then characteristic polynomial for A is
   a. $x^2 + 1$
   b. $x + 1$
   c. $x - 1$
   d. $x^2 - 1$

5. If a system of equations has zero solution, then
   a. Rank = number of variables
   b. Rank < number of variables
   c. Rank > number of variables
   d. None

6. Let T be linear transformation on R^2 into itself such that $T(1,0) = (1,2)$ and $T(1,1) = (0,2)$. Then $T(a,b)$ is equal to
   a. (a, 2b)
   b. (2a, b)
   c. (a-b, 2a)
   d. (a-b, 2b)

7. Z_7 is a vector space over Z_5.
   a. True
   b. False
   c. Can't be determined
   d. None

8. Let V be a finite dimensional space. T is a zero transformation on V. Then range of T is

a. V
b. $\{0\}$
c. ϕ
d. None

9. Let V be a vector space. T is a linear transformation of V into V such that $T(\alpha) = \alpha, \alpha \in V$ then T is

a. Identity transformation
b. Zero transformation
c. Invertible transformation
d. Orthogonal transformation

10. Which of the following is a 2-dimensional subspace of R^3 $over$ R

a. $\{(0,x,0) : x \in R\}$
b. $\{(0,x,0) : x \in R\} \cup \{(0,0,y) : y \in R\}$
c. $\{(x,y,0) : x, y \in R \; and \; x + y = 0\}$
d. $\{(0,x,z) : x, z \in R\}$

11. If I is an identity transformation on finite vector space V. Then nullity of I is

a. dimV
b. Zero
c. 1
d. None

12. If α is a characteristic root of a non-singular matrix A, then characteristic root of adj(A) is

a. $\alpha|A|$
b. α
c. $\frac{|A|}{\alpha}$
d. $\frac{|adjA|}{\alpha}$

13. Let P be a 4×4 matrix whose determinant is 10. The determinant of the matrix -3P is

a. -810
b. -30
c. 30
d. 810

14. If A and B are 3X3 real matrices such that rank (AB) = 1, then rank (BA) cannot be

a. 0
b. 1
c. 2
d. 3

15. Dimension of vector subspace $W = \{(a,b,c) : a = b = c\}$ $over\ vector\ space\ R^3(R)$ equals

a. 0
b. 1
c. 2
d. 3

16. An orthogonal set of non-zero vectors is

a. Linearly independent
b. Linearly dependent
c. Constant
d. None

17. The orthogonal complement of inner product space V is

a. Zero subspace
b. V itself
c. Any subspace
d. None

18. The Cauchy-Schwarz inequality states

a. $|<\alpha\beta>| \geq \|\alpha\|\|\beta\|$

b. $|<\alpha\beta>| \leq \|\alpha\|\|\beta\|$

c. $\|(\alpha+\beta)\| \geq \|\alpha\|\|\beta\|$

d. None

19. Let S be an orthonormal set then for $\alpha \in S$

a. $\|\alpha\| = 0$

b. $\|\alpha\| > 0$

c. $\|\alpha\| = 1$

d. $\|\alpha\| < 1$

20. Nullity of a characteristic matrix is

a. Number of non-zero rows

b. Number of zero rows

c. Rank of matrix

d. None

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Descriptive

Time : 2 hrs. 30 min. Marks:50

[Answer question no.1 & any four (4) from the rest]

1. Show that the matrix 10

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ is diagonalizable.

2. Define monic polynomial and minimal polynomial. Find the minimal 2+8=10

polynomial of the matrix $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$.

3. Prove that the vector space $V(F)$ is the direct sum of its subspaces 10

W_1 and W_2 over F if and only if

(i) $V = W_1 + W_2$

(ii) $W_1 \cap W_2 = \{0\}$

4. If T is an operator on R^3 whose basis is $B = \{(1,0,0),(0,1,0),(0,0,1)\}$ such that **10**

$[T:B] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$. Find the matrix of T with respect to basis

$B_1 = \{(0,1,-1),(1,-1,1),(-1,1,0)\}$

5. Using modal matrix, show that the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is diagonalizable. Hence find A^4. **10**

6. Let $T:V_3(R) \to V_3(R)$ be a transformation defined as $T(x,y,z) = (3x, x-y, 2x+y+z)$. Prove that T is invertible and find T^{-1}. Also, show that $(T^2 - I)(T - 3I) = O$. **5+5=10**

7. Define singular and non-singular transformations. Show that a linear transformation $T:U \to V$ is non-singular if and only if the set of images of linearly independent set is linearly independent. **2+8=10**

8. 1. State and prove Cauchy-Schwarz's inequality. **10**

<u>*Or*</u>

2. **a.** Let $T:V \to V$ be a linear operator and the characteristic polynomial $C(x) = (x+6)^4 (x-1)^3$ and minimal polynomial $m(x) = (x+6)^3 (x-1)^2$. Find the Jordan canonical form. **5+5=10**

b. Let $A = \begin{bmatrix} 0 & 0 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, then find its Jordan canonical form.

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