REV-01
MSM/25/30

2024/11

SET A

M.Sc. MATHEMATICS
THIRD SEMESTER
NUMBER THEORY
MSM – 302

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(Objective)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1×10=10***

1. The remainder obtained when $11^{104}+1$ is divided by 17is
   a. 1
   b. 0
   c. 7
   d. 12

2. Which of the following congruence has a solution
   a. $34x \equiv 5 (\text{mod } 12)$
   b. $54x \equiv 3 (\text{mod } 26)$
   c. $6x \equiv 15\ (\text{mod } 21)$
   d. $39x \equiv 1 (\text{mod } 13)$

3. The value of $1! + 2! + 3! + \cdots 100!\ (\text{mod } 12)$ is
   a. 9
   b. 0
   c. 6
   d. 3

4. Consider the congruence $x^2 \equiv a \text{mod } 64$. This congruence has a solution if
   a. $a = 6$
   b. $a = 11$
   c. $a = 13$
   d. $a = 17$

5. Which of the following equation have $[\overline{1;2}]$ as a continued fraction representation
   a. $2x^2 + 2x + 1 = 0$
   b. $2x^2 - 2x - 1 = 0$
   c. $2x^2 - 2x - 1 = 0$
   d. None of these

6. If 3 is a primitive root of p then the value p cannot be
   a. 17
   b. 29
   c. 23
   d. 31

7. If p is a factor of $2^{\frac{p-1}{2}} - 1$ then
   a. $p \equiv 3 (\text{mod } 8)$
   b. $p \equiv 5 (\text{mod } 8)$
   c. $p \equiv 1 (\text{mod } 8)$
   d. None of these

8. The value of $[1;\ 1, 1, 1, \cdots]$ is
   a. $\frac{F_{n+1}}{F_n}$ (F_n denotes the nth Fibonacci number)
   b. $\frac{F_n}{F_{n+1}}$ (F_n denotes the nth Fibonacci number)
   c. $\dfrac{1-\sqrt{5}}{2}$
   d. $\dfrac{1+\sqrt{5}}{2}$

9. Let $n > 1$ be an integer possessing primitive roots, then the value of n is equal to

a. 21
b. 26
c. 28
d. 32

10. The value of $1^{10} + 2^{10} + 3^{10} + \cdots + 10^{10} \pmod{11}$ is equal to

a. 0
b. 1
c. -1
d. None of these

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Descriptive

Time : 1 hr. 15 mins. **Marks: 25**

[Answer question no.1 & any two (2) from the rest]

1. Evaluate the continued fraction representation of $\sqrt{14}$. **5**

2. **a.** If $F = [a; b, a, b, \cdots]$ show that the roots of the equation $x^2 - (ab+2)x + 1 = 0$ are $1 + ab + aF$ and $1 - aF$. **6+4=10**

   b. Solve the common solution of the following system of congruences

$$2x \equiv 1 \pmod 5$$
$$3x \equiv 9 \pmod 6$$
$$4x \equiv 1 \pmod 7$$
$$5x \equiv 9 \pmod{11}$$

3. **a.** Show that if $(a, b) = 1$ then prove that **3+4+3 =10**

$$\gcd(a+b, a^2 - ab + b^2) = 1 \text{ or } 3.$$

   b. Solve the congruence $x^2 + 7x + 10 \equiv 0 \pmod{11}$.

   c. If p is an odd prime, prove that:

$$1^2 \cdot 3^2 \cdot 5^2 \cdots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod p$$

   and $2^2 \cdot 4^2 \cdot 6^2 \cdots (p-1)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod p$

4. **a.** Prove that- 2 is not a primitive root of any prime of the form$2^{2^n} + 1, n > 1$. **4+3+3 =10**

   b. Evaluate the Legendre symbol $\left(\frac{-46}{17}\right)$.

   c. For an odd prime p, prove that the congruence $2x^2 + 1 \equiv 0 \bmod p$ has a solution if and only if $p \equiv 1$ or $3 \pmod 8$.

5. **a.** If $C_k = \frac{p_k}{q_k}$ is the kth convergent of the finite simple continued fraction $[a_0; a_1, a_2, \cdots, a_n]$, then prove that 5+3+2 =10

$$p_k q_{k-1} - q_k p_{k-1} = (-1)^{k-1}, 1 \le k \le n$$

Also, prove that for $1 \le k \le n$, $\gcd(p_k, q_k) = 1$.

b. Prove that 3 is a quadratic nonresidue of all primes of the form $2^{2n} + 1$ for any n.

c. Find the remainder when $3^{24} \cdot 5^{13}$ is divided by 17 ?

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