2024/11

SET A

M.SC. MATHEMATICS
THIRD SEMESTER
ORDINARY DIFFERENTIAL EQQUATION II
MSM – 303

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. Full Marks: 35

(Objective)

Time: 15 mins. Marks: 10

Choose the correct answer from the following: ***1×10=10***

1. The nth approximation y_n of Picard's Method

   a. $y_n = y_1 - \int_{x_0}^{x} f(x, y_{n-2})dx$ b. $y_n = y_1 + \int_{x_0}^{x} f(x, y_{n-2})dx$

   c. $y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-2})dx$ d. $y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})dx$

2. Solution of the Differential Equation with single variable $\frac{dx}{dt} = \alpha x$ is

   a. $x(t) = ce^{-\alpha t}$ b. $x(t) = -ce^{\alpha t}$

   c. $x(t) = ce^{\alpha t}$ d. none of the above

3. In Existence and Uniqueness Theorem we consider a function $f(x, y)$ which is

   a. Continuous and $|f(x, y)| \geq M, M$ is constant

   b. Continuous and $|f(x, y)| \leq M, M$ is constant

   c. Continuous,Differentiable and $|f(x, y)| \geq M, M$ is constant

   d. Continuous,Differentiable and $|f(x, y)| \leq M, M$ is constant

4. The Auxillary equation of $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x$ is

   a. $m^2 - 5m + 6 = 0$ b. $m^2 - 4m + 6 = 0$

   c. $m^2 - 5m - 6 = 0$ d. $m^2 - 5m + 5 = 0$

5. Solution of $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

a. $\tan x \tan y = c, c$ is arbitrary constant

b. $\tan x = \tan yc, c$ is arbitrary constant

c. $\tan xc = \tan y, c$ is arbitrary constant

d. none of the above

6. Which of the following is an example of Strum-Liovelli's Problem

a. $\frac{d^3y}{dx^2} + \chi y = 0, y(0) = 0, y(\pi) = 0$

b. $\frac{d^2y}{dx^2} + \chi y = 0, y(0) = 0, y(\pi) = 0$

c. $\frac{dy}{dx} + \chi y = 0, y(0) = 0, y(\pi) = 0$

d. $\frac{d^4y}{dx^4} + \chi y = 0, y(0) = 0, y(\pi) = 0$

7. Lipschitz Condition is

a. $|f(x, y_1) - f(x, y_2)| \geq k|y_1 - y_2|$

b. $|f(x, y_1) - f(x, y_2)| \leq |y_1 - y_2|$

c. $|f(x, y_1) - f(x, y_2)| \leq k|y_1 + y_2|$

d. $|f(x, y_1) - f(x, y_2)| \leq k|y_1 - y_2|$

8. If we take a non-homogeneous DE $\left(\frac{d^2}{dx^2} + 2x^2\right)y(x) = f(x)$, then it can be written as $\hat{L}y(x) = f(x)$, where $\hat{L} = \left(\frac{d^2}{dx^2} + 2x^2\right)$. $\hat{L}^{-1}$ is known as

a. Green's function
b. Newton's function
c. Undetermined coefficient
d. Strum-Liovelli's function

9. The condition of singular solution for the equation of $y = px + \frac{a}{p}$ is

a. $4ax = 0$
b. $y^2 + 4ax = 0$
c. $y = 0$
d. P disc=C disc

10. Singular solution of the equation $9p^2(y-2)^2 = 4(3-y)$ is

a. $y = 3$ b. $y = -3$

c. $y = 2$ d. none of the above

-- --- --

(Descriptive)

Time : 1 hr. 15 mins. **Marks: 25**

[Answer question no.1 & any two (2) from the rest]

1. Find the third approximation of the solution of the equation $\frac{dy}{dx} = z, \frac{dz}{dx} = x^3(y+z)$ by Picard's method where $y = 1, z = 1/2$ where $x = 0$ **5**

2. a. Solve by Method of Undetermined coefficient. **6+4=10**

$$\frac{d^2y}{dx^2} - 4y = e^x + Sin2x$$

b. Solve

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^{-1}$$

3. State the Existence and Uniquenesss Theorem. Prove That the solution **2+8=10**

$$y(x) = y_0 + \int_{x_0}^{x} f[x, y(x)]dx$$

is exist.

4. 5+5=10

a. Show that for the problem $\frac{dy}{dx} = y, y(0) = 1$,the constant a in Picard's Theorem must be smaller than unity.

b. Give an example to show that there exist functions which do not ssatisfy the Lipschitz condition.

5. Find the Singular Solution of the follwoing 5+5=10

(a)

$$3y = 2px - \frac{2p^2}{x}$$

(b)

$$\sin px \cos y = \cos px \sin y + p$$

= = *** = =