REV-01
BMT/07/12

2024/11

B.SC. MATHEMATICS
FIRST SEMESTER
CALCULUS
BMT – 101

SET A

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. The number of possible functions that can be defined from $X = \{a, b\}$ to $Y = \{p, q, r\}$ is
   a. 8 b. 9
   c. 5 d. None of these

2. The number of one-one functions from $X = \{a, b\}$ to $Y = \{p, q, r\}$ is
   a. 4 b. 5
   c. 6 d. 7

3. The step function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = [x]$, where $[x]$ denotes the greatest integer $\leq x$ is
   a. Continuous for all $x \in \mathbb{R}$ b. Continuous at integers in $\mathbb{R}$
   c. Discontinuous at all integers in $\mathbb{R}$ d. Discontinuous at all non-integers in $\mathbb{R}$

4. The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{|x|}{x}$ is
   a. Continuous at $x = 0$ b. Discontinuous at $x = 0$
   c. Discontinuous at $x \neq 0$ d. False for all the cases here

5. Let $y = f(x) = |x|, x \in \mathbb{R}$. Then $x = 0$, $f(x)$ is
   a. Both continuous and derivable b. Continuous but not derivable
   c. Derivable but not continuous d. Neither continuous nor derivable

6. If $y = x \sin x$ then $\frac{d^2y}{dx^2}$ at $x = 0$ is
   a. 2 b. -2
   c. 0 d. 1

7. If $f(x) = \tan x$, then $f'(x)$ at $x = \frac{\pi}{4}$ is
   a. 2 b. $-\sqrt{2}$
   c. 2 d. -2

8. If $y = \sin x, x \in \mathbb{R}$, then y_n is
   a. $\cos\left(\frac{n\pi}{2} + x\right)$ b. $\sin\left(\frac{\pi}{2} + x\right)$
   c. $\cos\left(\frac{n\pi}{2} - x\right)$ d. $\sin\left(\frac{n\pi}{2} - x\right)$

9. The gradient of the tangent to the curve $y = x^3 - 2x$ at $x = 1$ is

a. 0
b. 1
c. -1
d. 2

10. The gradient of the normal to the curve $y = f(x)$ at a point x_0 on the curve is given by

a. $f'(x_0)$
b. $\frac{1}{f(x_0)}$
c. $-\frac{1}{f(x_0)}$
d. None of these

11. Let $y = f(x)$ be a function of real numbers such that

(i) $f(x)$ is continuous in $[a, b]$
(ii) $f(x)$ is derivable in (a, b)
(iii) $f(a) = f(b)$

Then by Rolle's theorem

a. $f'(x) > 0$ for all $x \in [a, b]$
b. $f'(x) < 0$ for all $x \in [a, b]$
c. $f'(x) = 0$ for at least one $x \in (a; b)$
d. None of these

12. The formula $\frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}}$ denotes the length of

a. Tangent
b. Normal
c. Subtangent
d. Subnormal at a point of the curve $y = f(x)$

13. If a function $y = f(x)$ is

(i) Continuous in $[a, b]$
(ii) Derivable in (a, b)

Then there is a point c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$ where

a. $a \leq c < b$
b. $a < c \leq b$
c. $a \leq c \leq b$
d. $a < c < b$

14. If $y = f(x) = \log x$, then y_n is given by

a. $\frac{1}{x^n}$
b. $\frac{(n-1)!}{x^n}$
c. $\frac{n!}{x^n}$
d. $(-1)^{n-1}\frac{(n-1)!}{x^n}$

15. The value of y_n, where $y = (ax + b)^n$ is

a. na^n
b. $n!\,a^n$
c. nab^n
d. $n!\,b^n$

16. Rolle's theorem is valid for the function $f(x)$ if

a. $f(x) = \tan x$ in $[0, \pi]$
b. $f(x) = \cos\frac{1}{x}$ in $[-1, 1]$
c. $f(x) = x^2$ in $[2, 3]$
d. $f(x) = x(x + 3)$ in $[-3, 0]$

17. The necessary condition for a function $f(x)$ to have an extreme value at $x = c$ is

a. $f'(c) > 0$
b. $f'(c) = 0$
c. $f'(c) < 0$
d. None of these

18. A function $f(x)$ has maximum value at $x = e$ if
a. $f'(c) = 0$ and $f''(c) > 0$
b. $f'(c) = 0$ and $f''(c) < 0$
c. $f'(c) = 0$ and $f''(c) \neq 0$
d. $f'(c) \neq 0$ and $f''(c) = 0$

19. If $f(x) = |x|$ then
a. $f'(0) = 0$
b. $f(x)$ is maximum at $x = 0$
c. $f(x)$ is minimum at $x = 0$
d. None of these

20. The function $f(x) = x^3 - 6x^2 + 24x + 4$ has
a. A maximum value at $x = 2$
b. A minimum value at $x = 2$
c. A maximum value at $x = 4$ and a minimum value at $x = 6$
d. Neither maximum nor minimum at any point.

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(Descriptive)

Time : 2 hrs. 30 min. **Marks: 50**

[Answer question no.1 & any four (4) from the rest]

1. **a.** Define derivative $f'(a)$ of a function $f(x)$ at a point $x = a$. Find $f'(a)$ for the function $f(x) = |x|$, if it exists. **5+5=10**

   b. If $y = f(x) = x^2 \sin x$ then compute $\left[\frac{dy}{dx}\right]_{x=0}$

2. **a.** Construct all possible functions from $A = \{a, b, c\}$ to $B = \{x, y\}$. **5+5=10**
   Hence find out
   (i) the number of onto functions
   (ii) the number of functions which are not onto.

   b. Consider the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = [x]$ where $[x]$ denotes the greatest integer $\leq x$.
   Show that f is continuous at $x = 0$ from the right but it is not continuous from the left.

3. **a.** What do you mean by $\lim_{x \to a} f(x) = l$. Show that $\lim_{x \to 0} \sin x = 0$. **5+5=10**

   b. A function $f: \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = |x| + |x - 1|, x \in \mathbb{R}$.
   Draw the graph of the function $f(x)$ for $-1 \leq x \leq 2$.

4. **a.** Examine the limit of the function $f(x)$ as $x \to 2$ where **5+5=10**

$$f(x) = \begin{cases} \dfrac{|x-2|}{x-2}, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

Examine the continuity of $f(x)$ at $x = 2$.

b. Evaluate the following limits

(i) $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$

(ii) $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$

5. **a.** Examine the continuity of the function $f : \mathbb{R} \to \mathbb{R}$ defined by **5+5=10**

$$f(x) = \begin{cases} x, & 0 \leq x < \dfrac{1}{2} \\ 1, & x = \dfrac{1}{2} \\ 1 - x, & \dfrac{1}{2} < x < 1 \end{cases}$$

b. If $y = \left(\frac{1}{x}\right)^x$, show that $y_2(1) = 0$ where

$$y_2(1) = \left[\frac{d^2y}{dx^2}\right]_{x=1}$$

6. **a.** If $y = (\sin^{-1} x)^2$, then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. **5+5=10**

Hence by using Leibnitz theorem show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$

b. If a function $y = f(x)$ is derivable at a point x_0 in its domain of definition, then prove that it is continuous at the point.

7. **a.** Find the equation of tangent to the parabola $y^2 = 4ax$ at $(a, -2a)$. **5+5=10**

b. State and prove Lagrange's mean value theorem.

8. **a.** Investigate the maximum and minimum values of the function **5+5=10**

$$f(x) = 2x^3 - 15x^2 + 36x + 10$$

b. Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$

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