REV-01
BSC/13/18

2024/11

SET A

B.SC. PHYSICS
FIRST SEMESTER
INTRODUCTION TO MATHEMATICAL PHYSICS
BSP – 101

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. Curl grad $\vec{F}$ = ?
   a. 1
   b. 0
   c. 3
   d. ∞

2. $\nabla \times (\phi \vec{a})$, where Φ is a scalar field and $\vec{a}$ is a vector field is given by
   a. $\phi \vec{\nabla} \times \vec{a}$
   b. $\nabla \vec{\phi} \times \vec{a}$
   c. $\vec{a} \times \nabla \vec{\phi}$
   d. $\nabla \vec{\phi} \times \vec{a} + \phi \vec{\nabla} \times \vec{a}$

3. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\nabla \times \nabla \times \vec{F}$ is
   a. 0
   b. $\hat{i}$
   c. $2\hat{j}$
   d. $3\hat{k}$

4. Which of the following is axial vector?
   a. momentum
   b. displacement
   c. torque
   d. force

5. What is a Laplacian operator?
   a. second order differential operator
   b. first order differential operator
   c. a complex operator
   d. null function

6. How to relate double integrals with line integrals in a plane
   a. Green's Theorem
   b. Stokes theorem
   c. Gauss theorem
   d. Greens hypothesis

7. The Wronskian of two differentiable functions f and g is
   a. W (f, g) = f g – g f '
   b. W(f, g) = f g' – g f
   c. W(f, g) = g g'– f f '
   d. W(f, g) = f g' – g f '

8. The magnitudes of two vectors are 3 and 4 units and their dot product is 6 units. The angle between the vectors is
   a. π/6
   b. π/3
   c. π/2
   d. π/4

9. -------- of gradient is a Laplacian operator. (Fill in the blank)

a. Divergence
b. Curl
c. Gradient
d. None of these

10. The gradient of a scalar quantity is a --------

a. vector function
b. vector field
c. Scalar function
d. Vector Potential

11. The electric field due to a point charge Q is expressed $\vec{E} = \frac{Q\hat{r}}{4\pi\varepsilon_0 r^2}$, then the divergence of electric field due to that point charge is

a. $\frac{3Q}{4\pi\varepsilon_0 r^2}$
b. $\frac{2Q}{4\pi\varepsilon_0 r}$
c. zero
d. $\frac{3Q}{4\pi\varepsilon_0 r}$

12. Stoke's theorem is the relationship between

a. Surface and volume integral
b. line and surface integral
c. line and volume integral
d. none of these

13. Let $\vec{F}$ be the force on a particle moving along C. Then $\int_C \vec{F} \cdot d\vec{r}$ represents

a. velocity of the particle
b. projection of $\vec{F}$ in the direction of the position vector of the particle
c. work done by the force
d. acceleration of the particle

14. What is the wronskian determinant of x^2, x^3

a. $2x^4$
b. x^4
c. $3\,x^4$
d. $4\,x^4$

15. What is a first-order differential equation?

a. An equation involving derivatives of any order
b. An equation involving only the first derivative of a function
c. An equation involving only the second derivative of a function
d. An equation involving no derivatives

16. Which of the following is a first-order differential equation?

a. $\frac{d^2y}{dx^2} + y = 0$
b. $\frac{dy}{dx} + y = 0$
c. $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + y = 0$
d. None of these

17. Particular integral of $y'' + 2y' - 3y = e^{2x}$ is

a. $-\frac{1}{5}e^{2x}$

b. $\frac{1}{5}e^{2x}$

c. $-\frac{1}{5}$

d. $\frac{1}{5}$

18. The roots of the characteristic equation of $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y = 0$

a. Real and equal

b. Real and but not equal

c. Imaginary

d. Undefined

19. For the right handed system of three coplanar vectors $\vec{A} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{B} = 3\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{C} = -\hat{i} + 4\hat{j} + m\hat{k}$, the value of m must be equal to

a. 0

b. 1

c. 6.5

d. 8

20. If $\vec{A} = x\hat{i}$ and $\vec{B} = y\hat{j}$ then $\nabla(\vec{A}.\vec{B})$ is equal to

a. $x\hat{i} + y\hat{j}$

b. $\frac{1}{2}yx^2\hat{i} + \frac{1}{2}xy^2\hat{j}$

c. 0

d. 2

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(Descriptive)

Time : 2 hrs. 30 min.

Marks: 50

[Answer question no.1 & any four (4) from the rest]

1. a. Find the value of $\nabla.(\frac{a\times r}{|r|^n})$, where r= $x\hat{i} + y\hat{j} + z\hat{k}$. 5+5=10

b. Solve $\frac{d^2y}{dx^2} + 6y = sin\,4x$

2. a. If four vectors whose position vectors are $\vec{a}, \vec{b}, \vec{c}$ are coplanar, show that $[\vec{a}\vec{b}\vec{c}] = [\vec{a}\vec{d}\vec{b}] + [\vec{a}\vec{d}\vec{c}] + [\vec{d}\vec{b}\vec{c}]$ 5+5=10

b. An equation relating to stability of an areoplane is

$$\frac{dv}{dt} = g\cos\alpha - kv,$$ where v is the velocity, g, α, k being constants. Find an expression for the velocity if $v=0$ when $t=0$.

3. State Green's theorem. (a) Using Green's theorem, evaluate $\int_c (x^2 y dx + x^2 dy)$, where c is the boundary described counter clockwise of the triangle with vertices (0,0), (1,0),(1,1). (b) A vector field **F** is given by **F**=sinyi+x(1+cosy)**j**. Evaluate the line integral $\int_c \mathbf{F}.d\mathbf{r}$, where c is the circular path given by $x^2 + y^2 = a^2$ — 2+4+4 =10

4. **a.** Evaluate $\nabla^2(\ln r)$. — 6+4=10

b. Prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$

5. State Gauss's Divergence theorem. Evaluate $\iint \vec{A} \cdot \hat{n} dS$, where $\vec{A} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. — 2+8=10

6. Solve — 5+5=10

a. $(D^2 - 4D + 4)Y = 2x^3 e^{2x}$

b. $(D^3 + 1)y = e^{-x}$

7. — 3+7=10

a. Prove that for every vector field $\vec{V}$, $div(curl\vec{V}) = 0$.

b. Show that $div(grad\ r^n) = n(n+1)r^{n-2}$, where $r=(x^2+y^2+z^2)^{1/2}$

8. Solve — 5+5=10

a. $(1+x^2)dy - xydx = 0$

b. Find the value of λ, for the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact.

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