REV-01
BSP/01/05

2024/07

SET A

B.Sc. PHYSICS
FIFTH SEMESTER
ADVANCED MATHEMATICAL PHYSICS
BSP – 504B [SPECIAL REPEAT]
[USE OMR SHEET FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. If W is a subspace of a vector space V then which of the following conditions is ture?
   a. $\vec{x}, \vec{y} \in W \Rightarrow \vec{x} + \vec{y} \in W$
   b. $\alpha \in W, \vec{x} \in W \Rightarrow \alpha\vec{x} \in W$
   c. $m(n\vec{x}) = (mn)\vec{x}, \forall \vec{x} \in W$, m and n are scalar.
   d. all of these

2. Which of the following is not a semi group?
   a. Set of natural numbers with respect to addition
   b. (N,*) where, m*n=l.c.m of m and n for all m, n belongs to N
   c. Set of integers with respect to division
   d. Set of real numbers with respect to subtraction

3. For every group G, the identity mapping I_G is
   a. A homomorphism of G onto itself
   b. An isomorphism of G onto itself
   c. zero
   d. None of the above

4. The set of vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}$ is said to be linearly dependent if $c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} = 0$, where c_1, c_2, c_3 are scalars, provided
   a. c_1 or c_2 or $c_3 \neq 0$
   b. $c_1, c_2, c_3 = 0$
   c. c_1, c_2, c_3 are even integers
   d. c_1, c_2, c_3 are positive real numbers

5. The set $\{1, , ^2\}$ is
   a. linearly independent
   b. linearly dependent
   c. linearly Span of P^2
   d. both 1 and 3

6. If A_k^{ij} is an antisymmetric tensor of rank 3 with respect to the indices i and j, which of the following is true?
   a. $A_k^{ij} = -A_i^{kj}$
   b. $A_k^{ij} = -A_k^{ji}$
   c. $A_k^{ij} = -A_j^{ik}$
   d. $A_i^{jk} = -A_i^{kj}$

7. If U(F) and V(F) are two vector spaces and T is a linear transformation, then the null space of T is defined as
   a. $N(T) = \{\alpha \in U: T(\alpha) = 0 \in V\}$
   b. $N(T) = \{\alpha \in U: T(\alpha) = e^{/} \in V\}$
   c. $N(T) = \{\beta \in V: T(\alpha) = \beta,$ for some $\alpha \in U\}$
   d. $N(T) = \{\alpha \in U: T(\alpha) = \alpha \in V$

8. The inner product of two mixed tensors A_ν^μ and $B_\gamma^{\alpha\beta}$ will produce a tensor of rank
   a. 5
   b. 3
   c. 2
   d. 1

9. Which of the following is the correct statement?

a. A scalar is a tensor of rank 0
b. A scalar is a tensor of rank 1
c. A scalar is a tensor of rank 2
d. A scalar is not a tensor

10. The Kronecker delta symbol is defined as

a. $\delta_\nu^\mu = \begin{cases} 0, \text{for } \mu, \nu = 0 \\ 1, \text{for } \mu, \nu = 1 \end{cases}$
b. $\delta_\nu^\mu = \begin{cases} 0, \text{for } \mu = \nu \\ 1, \text{for } \mu \neq \nu \end{cases}$
c. $\delta_\nu^\mu = \begin{cases} 0, \text{for } \mu \neq \nu \\ 1, \text{for } \mu = \nu \end{cases}$
d. $\delta_\nu^\mu = \begin{cases} 1, \text{for } \mu = \nu \\ -1, \text{for } \mu \neq \nu \end{cases}$

11. The modulus of each characteristic root of a unitary matrix is

a. Unity
b. 0
c. ∞
d. none of these

12. A square matrix A is idempotent if

a. $A' = A$
b. $A' = -A$
c. $A^2 = A$
d. $A^2 = I$

13. If a square matrix U such that $\overline{U} = U^{-1}$ then U is

a. Orthogonal
b. Unitary
c. Symmetric
d. Hermitian

14. If λ is an eigen value of a non-singular matrix A then the eigen value of A^{-1}

a. $\frac{1}{\lambda}$
b. λ
c. $-\lambda$
d. $-\frac{1}{\lambda}$

15. The sum of the eigen value of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

a. 2
b. 5
c. 7
d. 12

16. The Hamilton's canonical equation of motion in terms of Poisson's Bracket are

a. $\dot{q} = [q, H];\ \dot{p} = [p, H]$
b. $\dot{q} = [p, H];\ \dot{p} = [q, H]$
c. $\dot{q} = [H, q];\ \dot{p} = [H, p]$
d. $\dot{q} = [H, p];\ \dot{p} = [H, q]$

17. The Lagrangian equation of motion are __________ order differential equations.

a. First
b. Second
c. Zero
d. Fourth

18. The generalized coordinate has the dimension of velocity, generalize velocity has the dimensions of

a. displacement
b. Velocity
c. acceleration
d. force

19. The generalized coordinates for motion of a particle moving on the surface of a sphere of radius 'a' are ________.

a. a *and* θ
b. a *and* $\emptyset$
c. θ *and* $\emptyset$
d. 0 *and* $\emptyset$

20. Poisson bracket are ____________ under canonical transformation

a. Invariant
b. Variant
c. Equivalent to
d. None of these

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Descriptive

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Using Cayley-Hamilton Theorem calculate A^4 for the following matrix 5+5=10

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

b. If $A_\sigma^{\mu\nu}$ and $B_\sigma^{\mu\nu}$ are two mixed tensors of rank 3, then prove that the addition and subtraction of $A_\sigma^{\mu\nu}$ and $B_\sigma^{\mu\nu}$ are also tensors of same rank and same type.

2. a. Find Lagrange's equation of motion for an electrical circuit comprising an inductance L and capacitance C. 5+5=10
The capacitor is charged to q coulombs and current flowing in the circuit is i amperes.

b. Obtain the equation of motion of two masses, connected by an inextensible string passing over a small smooth pulley.

3. a. Find the relation between Poisson bracket and angular momentum. 5+5=10

b. If $[\phi,\psi]$ be the Poisson bracket, then prove that, $\frac{\partial}{\partial t}[\phi,\psi]=[\frac{\partial \phi}{\partial t}, \psi]+[\phi, \frac{\partial \psi}{\partial t}]$.

4. a. Using Euler's equation, prove that the shortest distance between two points in a plane is a straight line. 5+2+3 =10

b. Prove that the intersection of two subspaces of a vector space is also a subspace.

c. Show that $U = \{a + bx + cx^2 \in P_2 | a = b = c\}$ is a subspace of P_2

5. a. A homomorphism f defined from a group G to G' is an isomorphism if $ker(f) = \{e\}$. 4+3+3 =10

b. If $f : R \to R$ be defined by $f(x) = -7x$, check if f is a homomorphism or not.

c. Show that the additive group (R,+) of real numbers is isomorphic to the multiplicative group $(R^{+},\times)$ of positive real numbers.

6. a. What do you mean by kernel of linear transformation? Define range and null space of a linear transformation. 2+3+5 =10

b. If $C(R)$ be the vector space of real functions and the map is defined by $T(f(x)) = (f(x))^2$ for $f(x) \in C(R)$. Determine if T is a linear transformation or not.

7. a. Show that any tensor of rank 2 can be expressed as a sum of a symmetric tensor and an antisymmetric tensor, each of rank 2. 4+3+3 =10

b. Prove that a collection of vectors containing null vectors linearly dependent.

c. Check whether the following set of vectors are linearly dependent or independent:

Span of $\left\{\begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}4\\6\end{bmatrix}\right\}$

8. Solve the differential equation by matrix method. 10

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 0,\ x(0) = 1,\ x'(0) = 2$$

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