

B.Sc. PHYSICS  
SIXTH SEMESTER  
MATHEMATICAL PHYSICS  
BSP – 603A [SPECIAL REPEAT]  
[USE OMR FOR OBJECTIVE PART]

**SET  
A**

Duration: 3 hrs.

Full Marks: 70

(Objective)

Time: 30 min.

Marks: 20

1×20=20

Choose the correct answer from the following:

- The magnitude of the complex number  $\frac{1}{\sqrt{2}}(-1 + i)$  is  
a. -1  
b. 1  
c. 0  
d. 2
- The value of  $i^9$  is  
a. 1  
b.  $i$   
c.  $-i$   
d. -1
- The magnitude of the difference of the complex numbers  $(4 + 2i)$  and  $(5 + 3i)$  is  
a.  $\sqrt{2}$   
b. 1  
c. 2  
d. 0
- The reciprocal of the complex number  $(4 - 3i)$  is  
a.  $\frac{1}{25}(-4 - 3i)$   
b.  $\frac{1}{25}(-4 + 3i)$   
c.  $\frac{1}{25}(4 - 3i)$   
d.  $\frac{1}{25}(4 + 3i)$
- The Laplace transform  $f(t) = t^0$  is  
a.  $s^{-1}$   
b.  $s^{-2}$   
c.  $s^1$   
d.  $s^2$
- If  $L[f(x)] = F(s)$  then  $L[\cos 2x]$  is  
a.  $\frac{2s}{s^2+4}$   
b.  $\frac{2s}{s^2-4}$   
c.  $\frac{2s}{s^2+2}$   
d.  $\frac{s}{s^2+2}$
- If  $L[F(x)] = f(s)$  then  $L[x^n F(x)]$  is  
a.  $(-1)^n f^{n+1}(s)$   
b.  $(-1)^n f^{n-1}(s)$   
c.  $(-1)^n f^n(s)$   
d.  $(-1)^n f^{-n+1}(s)$
- $\frac{1}{3!} L^{-1}(1/s^4)$  is  
a.  $x^1$   
b.  $x^2$   
c.  $x^3$   
d.  $x^4$
- The value of  $\oint \frac{e^z}{z+1} dz$  for a circle of  $|z|=1$  is  
a.  $2\pi i$   
b.  $\pi i$   
c. 0  
d. 1

10. The value of  $\oint \frac{dz}{z-a}$  for a circle of  $|z-a|=r$  is
- 0
  - $2\pi i$
  - $\pi i$
  - $2\pi$
11. What are the conditions called which are required for a signal to fulfill to be represented as Fourier series?
- Dirichlet's conditions
  - Gibbs phenomenon
  - Fourier conditions
  - Fourier phenomenon
12. Choose the condition from below that is not a part of Dirichlet's conditions?
- It is single-valued, if the function  $f(x)$  for the interval  $(-\pi, \pi)$
  - It is bounded, if the function  $f(x)$  for the interval  $(-\pi, \pi)$
  - It has only a finite number of discontinuous, if the function  $f(x)$  for the interval  $(-\pi, \pi)$
  - It is a periodic signal, if the function  $f(x)$  for the interval  $(-\pi, \pi)$
13. For the given periodic function  $f(x) = x^3$  for  $-\pi < x < \pi$  the coefficient  $a_0$  is
- 6.8968
  - 6.8968
  - 0
  - 0.7468
14. A function  $f(x)$  is called skew symmetric function if
- $f(-x) = -f(x)$
  - $f(-x) = f(x)$
  - $f(-x) = -f(-x)$
  - $f(-x) = 0$
15. If the Fourier series of  $f(x)$  has only cosine terms then  $f(x)$  must be
- Odd function
  - Even function
  - Fundamental harmonic
  - Second harmonic
16. Which of the following is the Fourier sine transform of  $f(x)$ ?
- $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \sin(st) dt$
  - $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F(s) \sin(sx) ds$
  - $F_s[f(x)] = \sqrt{\frac{\pi}{2}} \int_0^{\infty} F(s) \sin(sx) dx$
  - $F_s[f(x)] = \sqrt{\frac{\pi}{2}} \int_0^{\infty} f(t) \sin(st) dt$
17. At what condition  $F\{f'(x)\} = isF(s)$
- $f(x) \rightarrow 0$  as  $x \rightarrow \pm\infty$
  - $f(x) \rightarrow \infty$  as  $x \rightarrow 0$
  - $f(x) \rightarrow \infty$  as  $x \rightarrow \pm\infty$
  - none of these

18. Fill in the blank. The property is known as-----, when  $F(s)$  is the complex Fourier transform of  $f(x)$  then  $F\{f(x-a)\} = e^{isa} F(s)$

- a. Shifting property
- b. Change of scale property
- c. Linear property
- d. Modulation theorem

19.  $F\{f''(x)\} = ?$

- a.  $(-is)^n F(s)$
- b.  $(is)^n F(s)$
- c.  $isF(s)$
- d.  $(is)^n F''(s)$

20. Fourier transform of  $f(t)$ -----x Laplace transform of  $g(t)$ . Fill in the blank

- a.  $\frac{1}{\sqrt{2\pi}}$
- b.  $\frac{1}{\sqrt{2\pi}}$
- c.  $\frac{1}{\sqrt{\pi}}$
- d. None of these

**( Descriptive )**

Time : 2 hrs. 30 mins.

Marks : 50

*[ Answer question no.1 & any four (4) from the rest ]*

1. a. Solution the differential equation  $\frac{d^2y}{dt^2} + y(t) = t$  using Laplace transform method where the boundary conditions are  $y(t=0) = 0$  and  $y'(t=0) = 1$ . 7+3=10
- b. The Laplace transform of  $ax^2 + bx^3$  will be?
2. a. Write the Dirichlet's condition for a Fourier series. 2+3+3+2=10
- b. Determine the Fourier coefficient  $a_0$  and  $a_n$ .
- c. What do you mean by fundamental harmonic of Fourier series.
3. a. If  $n$  is a positive integer, prove that  $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos \frac{n\pi}{4}$ . 4+3+3=10
- b. If  $\omega$  is a cube root of unity then find  $(1+\omega)^6$ .
- c. Express  $\frac{1}{2}e^{\frac{i\pi}{3}}$  in the complex number  $a + ib$ .
4. What do you mean by even function of a Fourier series? Expand the function  $f(x) = x \sin x$ , as a Fourier series in the interval  $-\pi < x < \pi$ . Hence deduce that  $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots = \frac{\pi-2}{4}$ . 1+5+4=10
5. a. Using Cauchy's integral formula evaluate  $\oint \frac{z}{z^2-3z+2} dz$  for a circle  $|z-2|=1/2$ . 4+3+3=10
- b. Find the inverse transform of  $\frac{2}{(s-2)^2+4}$ .
- c. Find the Laplace transform  $L[F(t)]$  if

$$F(t) = \begin{cases} \cos(t - \frac{\pi}{4}), & t > \frac{\pi}{4} \\ 0, & t < \frac{\pi}{4} \end{cases}$$

6. a. If  $F_c(s) = \frac{1}{2} \tan^{-1}\left(\frac{2}{s^2}\right)$ , find  $f(x)$ .

6+4=10

b. Establish the relationship between Fourier and Laplace transforms.

7. a. Find the value of a & b if  $\frac{(1+i)a-2i}{3+i} + \frac{(2-3i)b+i}{3-i} = i$ .

4+2+4  
=10

b. Find the value of  $\sqrt{i} + \sqrt{-i}$ .

c. Find the complex number if  $\arg(z+1) = \frac{\pi}{6}$  and  $\arg(z-1) = \frac{2\pi}{3}$ .

5+5=10

8. Find Fourier sine and cosine transform of  $x^{n-1}$ .

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