REV-01
BSP/01/05

2024/07

SET A

B.Sc. PHYSICS
THIRD SEMESTER
MATHEMATICAL PHYSICS II
BSP – 301 [SPECIAL REPEAT]

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. The Rodrigue formula for Legendre Polynomial $P_n(x)$ is given by

   a. $P_n(x) = \frac{n!}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$
   b. $P_n(x) = \frac{n!}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^{n-1}$
   c. $P_n(x) = \frac{n!}{2^{n-1}}\frac{d^n}{dx^n}(x^2-1)^n$
   d. $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$

2. The polynomial $2x^2+x+3$ in terms of Legendre's polynomial is

   a. $\frac{1}{3}[4P_2-3P_1+11P_0]$
   b. $\frac{1}{3}[4P_2+3P_1-11P_0]$
   c. $\frac{1}{3}[4P_2+3P_1+11P_0]$
   d. $\frac{1}{3}[4P_2-3P_1-11P_0]$

3. If A and B are the matrices of same order such that $AB = A$ and $BA = B$, then A and B are

   a. Nilpotent
   b. Idempotent
   c. Singular
   d. Hermitian

4. If $P_n(x)$ and $Q_n(x)$ are two independent solution of Legendre equation then the general solution is

   a. $y = AP_n(x)+BQ_n(x)$,
   b. $y = AP_n^2(x)+BQ_n(x)$
   c. $y = AP_n^2(x)+BQ_n^2(x)$
   d. None of these

5. If x=0 is a regular singular point of the differential equation $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$ and m_1, m_2 are real and different roots then

   a. $y=c_1(y)_{m_1}+c_2(y)_{m_2}$
   b. $y=c_1(y)_{m_1}+c_2(\frac{dy}{dm})_{m_1}$

c. $y = c_1(y)_{m_1}$

d. $y = c_1(y)_{m_1} + c_2(y)_{m_1}$

6. The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2} = \sin(xy)$

a. $z = -x^{-2}\sin(xy) + xf(x) + g(x)$

b. $z = -x^2\sin(xy) - yf(x) + g(x)$

c. $z = -y^2\sin(xy) + yf(x) + g(x)$

d. $z = -x^2\sin(xy) + yf(x) + g(x)$

7. A partial differential equation has

a. one independent variable

b. two or more independent variables

c. more than one dependent variable

d. equal number of dependent and independent variables

8. How many constants are require to make a 2nd order partial differential equation

a. 3

b. 2

c. 1

d. 0

9. Complete solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$ is

a. $z = f_1(y+2x) + xf_2(y+2x)$

b. $z = f_1(2y+x) + xf_2(2y+x)$

c. $z = f_1(y+x+2) + xf_2(y+x+2)$

d. $z = f_1(y-2x) + xf_2(y-2x)$

10. What is the value of $\beta(z,1)$

a. $\frac{1}{z}$

b. $\frac{1}{z+1}$

c. $\frac{1}{z(z+1)}$

d. $\frac{1}{z-1}$

11. $\frac{\Gamma(-1/2)}{\Gamma(1/2)}$

a. 2

b. -2

c. $\frac{1}{2}$

d. $-\frac{1}{2}$

12. If $1.3.5.......(2n-1) = \frac{2^n}{\sqrt{\pi}}\Gamma(p)$, then p is

a. $n + \frac{2}{3}$

b. $n + \frac{1}{2}$

c. $n+\frac{1}{3}$

d. $\frac{n}{2}-1$

13. $\int_0^\infty e^{-t^2}\,dt = ?$

a. $\sqrt{\pi}$

b. $\frac{\sqrt{\pi}}{2}$

c. π

d. 0

14. The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigen value is

a. -1

b. 0

c. 2

d. 1

15. The matrix A is defined as $A=\begin{bmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & 2\end{bmatrix}$. The eigenvalues of A^2 are

a. -1, -9, -4

b. 1, 9, 4

c. -1, -3, 2

d. None of these

16. Find the sum and product of the eigen value of the matrix $\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$

a. 7, 5

b. 9, 5

c. 7, 10

d. 9, 27

17. x= a is called an ordinary point of the differential equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$, when

a. $P=\infty, Q\neq\infty$

b. $P\neq\infty, Q\neq\infty$

c. $P=\infty, Q=\infty$

d. $P\neq\infty, Q=\infty$

18. If we interchange the rows and column of a matrix then the new matrix obtained is known as

a. symmetric

b. square

c. transpose

d. triangular

19. A square matrix A is called an unitary matrix if

a. $A^\theta A = I$

b. $\frac{A^\theta}{A} = I$

c. $A^\theta = I$

d. $A^\theta A = 0$

20. Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is

a. 2
b. 1
c. 3
d. 0

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Descriptive

Time : 2 hrs. 30 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. i. Form the partial differential equation from $z=(x+a)(y+b)$ 1+5+4 =10

ii. By method of separation of variables solve the partial differential equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.

iii. Solve $\frac{\partial^3 z}{\partial x^2 \partial y}=\cos(2x+3y)$

2. i. Find the deflection $u(x,y,t)$ of the square membrane with $a=b=c=1$, if the initial velocity is zero and the initial deflection $f(x,y)=A\sin \pi x \sin 2\pi y$. 10

3. 5+3+2 =10

i. Prove that $\int_0^1 \frac{x^2}{\sqrt{(1-x^4)}}dx \times \int_0^1 \frac{dx}{\sqrt{(1+x^4)}}=\frac{\pi}{4\sqrt{2}}$

ii. Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n}{(m+1)^{n+1}}\Gamma(n+1)$

iii. Evaluate $\Gamma\left(-\frac{1}{2}\right)$

4. i. Using Frobenius methods solve $x(x-1)y'' + (3x-1)y' + y = 0$. 7+3=10

ii. Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre Polynomials.

5. 1+3+3+3 =10

(i) Define orthogonal matrix.

(ii) Verify that $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.

(iii) Determine the values of α, β, γ when $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal.

(iv) Prove that $(AB)^n = A^n.B^n$, if $A.B = B.A$

6. (i) Show that any square matrix can be expressed as the sum of two matrices, one symmetric and other anti-symmetric. 2+4+4 =10

(ii) The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. Find the eigen values of $3A^3 + 5A^2 - 6A + 2I$ where I is the unit matrix of order 3.

(iii) Prove that if the product of two matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is zero then θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

7.	i.	Evaluate Beta function in terms of gamma function.	**4+4+2 =10**
	ii.	Prove that $1.3.5....(2n-1)=\dfrac{2^n\,\Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}}$	

Show that $\beta(l,m)=\beta(m,l)$.

8.	(i)	Find regular singular points of the differential equation $x(x-2)^2y''+2(x-2)y'+(x+3)y=0$.	**3+3+4 =10**
	(ii)	Draw and explain the graph for Legendre's polynomial P_0, P_1, P_2, P_3, P_4.	
	(iii)	Solve $\dfrac{\partial^3 z}{\partial x^2\partial y}=\cos(2x+3y)$	

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