REV-01
MSP/17/22

2024/05

SET A

M.Sc. PHYSICS
SECOND SEMESTER
MATHEMATICAL PHYSICS-I
MSP – 201

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. If the Laplace transform of $F(x)$ is $f(s)$ $(i.e. L[F(x)] = f(s))$, then $L\left[e^{ax}\sin(ax)\right]$ is
   a. $\dfrac{a}{(s-a)^2+a^2}$
   b. $\dfrac{a}{(s-a)^2-a^2}$
   c. $\dfrac{s}{(s-a)^2+a^2}$
   d. $\dfrac{s}{(s-a)^2-a^2}$

2. If $(1-2\,xh+h^2)^{-1/2} = \sum h^n p_n(x)$ then $p_3(-1)$ will be
   a. -1
   b. 1
   c. 0
   d. -2

3. Inverse Laplace transform of $\dfrac{s}{s^2+a^2}$ is
   a. $\cos at$
   b. $\cosh at$
   c. $\sinh at$
   d. $\sin at$

4. The value of $H_1\left(-\frac{1}{2}\right)$ will be
   a. 1
   b. 2
   c. 0
   d. -1

5. If $\{F(t)\} = \bar{f}(s)$, then $L\{\int_0^t F(x)dx\}$ is
   a. $\int_0^s \bar{f}(s)ds$
   b. $\int_0^s \frac{1}{s}\bar{f}(s)ds$
   c. $\frac{1}{s}\bar{f}(s)ds$
   d. $s\bar{f}(s)ds$

6. The coefficient of h^1 in the function $(1-2xh+h^2)^{-1/2}$ will be

a. x
b. $-x$
c. $-2x$
d. $x/2$

7. Fourier cosine transform of $\dfrac{1}{\sqrt{x}}$

a. $\sqrt{\dfrac{\pi}{2s}}$
b. $\sqrt{\dfrac{\pi}{s}}$
c. $\sqrt{\dfrac{2\pi}{s}}$
d. $\dfrac{1}{\sqrt{s}}$

8. The Bessel function of first kind $J_{\frac{1}{2}}\left(\frac{\pi}{2}\right)$ will be

a. $\frac{\pi}{2}$
b. $\frac{\pi}{4}$
c. $\frac{2}{\pi}$
d. $\frac{4}{\pi}$

9. $F_s[f(ax)]=?$

a. $\frac{1}{a}F_s(\frac{s}{a})$
b. $F_s(\frac{s}{a})$
c. $\frac{1}{a}F_s(sa)$
d. $\frac{s}{a}F_s(\frac{s}{a})$

10. The values of $L_1\left(\frac{3}{2}\right)$ will be

a. $\frac{1}{2}$
b. $-\frac{1}{2}$
c. $\frac{3}{2}$
d. 0

11. Find the dummy index in δ_{im}^{ijk}

a. j
b. k
c. i
d. m

12. The expression $H_1(x)-2x\,H_0(x)$ in terms of x will be

a. $2x$
b. $x-2$
c. 0
d. $2x-2$

13. $\delta_j^i \delta_k^j=?$

a. δ_j^i
b. δ_k^i
c. δ_i^k
d. δ_i^i

14. The value of a in the function $f(x) = 2x + 1 = a\, p_1(x) + p_0(x)$ will be

a. 2
b. -2
c. -1
d. 1

15. Write the following using summation convention

$(x^1)^2 + (x^2)^2 + (x^3)^2 + \dots\dots\dots\dots + (x^n)^2$

a. $(x^i)^2$
b. $(x^i)^i$
c. $(x^1)^i$
d. 0

16. The trigonometric function $\cos x$ in terms of Bessel polynomial will be

a. $J_0(x) + 2J_2(x) + 2J_4(x) + 2J_6(x) + \cdots$
b. $J_1(x) - 2J_3(x) + 2J_5(x) - 2J_7(x) + \cdots$
c. $J_1(x) + 2J_3(x) + 2J_5(x) + 2J_7(x) + \cdots$
d. $J_0(x) - 2J_2(x) + 2J_4(x) - 2J_6(x) + \cdots$.

17. 0^{th} rank tensor is known as

a. Vector
b. Scalar
c. Covariant
d. Contravariant

18. The value of a in the function $f(x) = x + 1 = H_0(x) + a\, H_1(x)$

a. 1
b. $-\frac{1}{2}$
c. $\frac{1}{2}$
d. -1

19. Which type of tensor addition and subtraction is possible?

a. Contravariant tensor
b. Covariant tensor
c. Mixed tensor
d. Same ranked tensor

20. The value of a in the function $f(x) = -x = a\, L_0(x) + L_1(x)$

a. -1
b. 0
c. 2
d. 1

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. What is Bessels' equation? Obtain a polynomial solution $J_n(x)$ of the Bessel's equation. **2+8=10**

2. Solve the following equation by Laplace transform **10**

   $y''' - 2y'' + 5y' = 0$; $y = 0$, $y' = 1$ at t=0and $y = 1$ at $t = \frac{\pi}{8}$.

3. a. Find Hermite polynomial expression $H_3(x)$ using Rodrigue's formula. **3+4+3 =10**

   b. Express the function $f(x) = 2x^2 - x + 4$ in terms of Hermite polynomial.

   c. Show that Hermite polynomial $H_n(x)$ satisfies the following recurrence relation: $2n\, H_{n-1}(x) = H'_n(x)$.

4. a. Obtain Laplace transform of derivative for order 'n'. **5+5=10**

   b. Find the Laplace transform of $F(t) = \begin{cases} 1, & 0 \le t < 1 \\ t, & 1 \le t < 2 \\ t^2 & 2 \le t < \infty \end{cases}$

5. a. Define Voltera Integral equation (VIE) of the first and second kind. **4+6=10**

   b. Reduce the boundary value problem to the Voltera Integral equation:

   $y''(x) + x\, y(x) = 1$, where $y(0) = 0$ and $y'(0) = 1$.

6. a. If $a_{\alpha\beta} x^\alpha x^\beta$=0 for all values of the variables $x^1, x^2, x^3 \ldots\ldots\ldots x^n$, then show that $a_{\mu\vartheta} + a_{\vartheta\mu} = 0$ **5+5=10**

b. If A^{μ} and B_{υ} are the components of a contravariant and covariant tensors of rank one, show that $C^{\mu}_{\upsilon} = A^{\mu} B_{\upsilon}$ are the components of a mixed tensor of rank two.

7. **a.** Show that Legendre polynomial $p_n(x)$ satisfies the following orthogonality condition **6+4=10**

$$\int_{-1}^{1} p_n(x)\dot{p}_m(x)dx = 0\ , n \neq m.$$

b. Express $p_2(x) + 2p_1(x) - 3p_0(x)$ in terms of a polynomial function x.

8. **a.** Find the Fourier transform of the function **5+5=10**

$$f(x) = \{ \begin{matrix} 1+\frac{x}{a} & for - a < x < 0 \\ 1-\frac{x}{a} & for\ 0 < x < a \\ 0 & otherwise \end{matrix}$$

b. Find the Fourier sine transformation of $\frac{1}{x}$.

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