REV-01
BSM/14/19

2024/06

SET A

B.Sc. MATHEMATICS
FOURTH SEMESTER
COMPLEX ANALYSIS
BSM – 941 IDMJ

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. If $z_1 = 3 + 2i$ and $z_2 = -7 - i$ then $z_1 + z_2$ will be
   a. $-5 + 2i$
   b. $2 - 5i$
   c. $-4 + i$
   d. None of these

2. If $z_1 = 8 - 6i$ and $z_2 = 2i - 7$ then $z_1 - z_2$ will be
   a. $6 + i$
   b. $15 - 8i$
   c. $4 - i$
   d. None of these

3. If $z_1 = 4 + 2i$ and $z_2 = 2 - 3i$ the $z_1 z_2$ will be
   a. $14 - 8i$
   b. $14 + 8i$
   c. $8 + 14i$
   d. $8 - 14i$

4. If $z_1 = 2 + i$ and $z_2 = 3 - 2i$ then $\frac{z_1}{z_2}$ will be
   a. $\frac{13}{4} + i\,\frac{13}{7}$
   b. $\frac{4}{13} + i\,\frac{7}{13}$
   c. $\frac{7}{13} + i\,\frac{4}{13}$
   d. None of these

5. Given the magnitude $r = 2$ and amplitude $\theta = \frac{\pi}{3}$ of a complex number $z = x + iy$, z will be given by
   a. $1 - \sqrt{3}\,i$
   b. $\sqrt{3} - i$
   c. $\sqrt{3} + i$
   d. $1 + \sqrt{3}\,i$

6. If $z = 3 - 4i$ then $|z|$ is
   a. 3
   b. 4
   c. 5
   d. -5

7. For two complex numbers z_1 and z_2, $|z_1 + z_2|$ is
   a. equal to $|z_1| + |z_2|$
   b. greater than $|z_1| + |z_2|$
   c. less than $|z_1| + |z_2|$
   d. Both (a) and (c)

8. For $w = f(z) = \frac{1}{z}$, if $z = 1 + i$ then w is
   a. $\frac{1}{2} + \frac{i}{2}$
   b. $\frac{1}{2} - \frac{i}{2}$
   c. $1 - \frac{i}{2}$
   d. $1 + \frac{i}{2}$

9. Let $w = f(z) = az$ where $= 2 + 3i$. Then $z = -i$ is mapped to w, where w is

a. $3 + 2i$ b. $3 - 2i$
c. $-3 + 2i$ d. $-3 - 2i$

10. Let $w = f(z) = az^2 + bz + c$ where $a = i, b = 1, c = -1$. Then $z = i$ is mapped to w, where w is

a. 1 b. -1
c. 2 d. -2

11. If $f(z) = \frac{1}{z}$ then $f'(i)$ is equal to

a. 1 b. -1
c. 0 d. None of these

12. For an analytic complex function $w = f(z) = u + iv$ the Cauchy-Riemann equations are

a. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ b. $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
c. Both (a) and (b) d. None of (a) and (b)

13. If $f(z) = 2z^3 - 1$ then $f'(1 - i)$ is

a. $6i$ b. $-6i$
c. $12i$ d. $-12i$

14. If $f(z) = u + iv$ is analytic then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is

a. ≥ 0 b. ≤ 0
c. $= 0$ d. None of these

15. Let $f(z)$ be a complex function with
(i) $\lim f(z) = l_1$ as $z \to 0$ along the real axis
(ii) $\lim f(z) = l_2$ as $z \to 0$ along the imaginary axis.
If $\lim_{z \to 0} f(z)$ exists then

a. $l_1 \geq l_2$ b. $l_1 = l_2$
c. $l_1 \leq l_2$ d. None of these

16. Given any complex number z and any real number θ, $e^{i\theta}z$ geometrically means

a. rotation of z through θ in anticlockwise sense b. rotation of z through θ inclockwise sense
c. change in magnitude of z without rotation. d. None of these

17. Let $w = f(z)$ be analytic in a domain R of the complex plane. If C_1 and C_2 be two paths in R joining any two points a and b with $\oint_{C_1} f(z)dz = l_1$, $\oint_{C_2} f(z)dz = l_2$ then

a. $l_1 \neq l_2$ b. $l_1 = l_2$
c. $l_1 = -l_2$ d. None of these

18. Let $w = f(z)$ be analytic with $f'(z)$ continuous in a domain R of the complex plane. For any simple closed curve C, $\oint_C f(z)dz$ is

a. equal to 0
b. less than 0
c. greater than 0
d. None of these

19. Number of roots of $z^7 - 1 = 0$ is

a. less than 7
b. greater than 7
c. equal to 7
d. None of these

20. Conjugate of $\frac{1}{i}$ is

a. i
b. $-i$
c. $2i$
d. None of these

-- --- --

Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. Let z_1 and z_2 denote the complex numbers $2 + i$ and $3 - 2i$ respectively. Find two real numbers x and y if **2+2+2+4 =10**

(i) $z_1 z_2 = x + iy$

(ii) $\frac{z_1}{z_2} = x + iy$

Hence plot $z_1, z_2, z_1 z_2$ and $\frac{z_1}{z_2}$ on the complex plane.

b. Given a complex number z interpret $ze^{i\alpha}$ geometrically, where α is real.

2. a. Evaluate **3+3+4=10**

(i) $\lim_{z \to 1+i} z^2 - 5z + 10$

(ii) $\lim_{z \to 2+i} \frac{(2z+3)(z-1)}{z^2-2z+4}$

b. Using the property of complex numbers find out all the fifth roots of 1.

3. **a.** Let $w = f(z) = z^2$ where z is a complex number of the form $z = x + iy$, x and y being real. Find the values of $w = u + iv$ corresponding to — 2+2+2+2 +2 =10

(i) $z = -2 + i$

(ii) $z = 1 - 3i$

Also demonstrate graphically the correspondence of points from z-plane to w-plane.

b. Find $\lim_{z \to 0} \frac{\bar{z}}{z}$ when (i) $z \to 0$ along real axis.

(ii) $z \to 0$ along imaginary axis

Hence show that $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exist.

4. **a.** If $w = f(z)$ be a function of complex numbers defined in a domain D then define the derivatives $f'(z)$ at $z = z_0 \in D$. Hence find $f'(z_0)$ if $f(z) = z^3 - 2z$. — 5+5=10

b. When is a function $w = f(z)$ said to be analytic? Show that $f(z) = \bar{z}$ is nowhere analytic.

5. **a.** If $f(z) = u + iv$ be analytic then show that the real function $u = u(x, y)$ and $v = v(x, y)$ satisfy Cauchy Riemann equations i.e. — 5+5=10

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

b. What are harmonic functions? Show that the real and imaginary parts u and v of an analytic function $f(z) = u + iv$ are harmonic?

6. **a.** Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic. — 5+5=10

b. In reference to part (a) of the question, find a function v such that $f(z) = u + iv$ becomes an analytic function.

7. a. Let $f(z)$ be continuous at all points of a rectifiable curve C, define line integral $\int_a^b f(z)\ dz$ for $f(z)$ from a point a to another point b on C. 5+5=10

b. Evaluate the line integral $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the straight lines from $(0, 3)$ to $(2, 3)$ and then from $(2, 3)$ to $(2, 4)$.

8. a. If $f(z)$ is analytic in a simply connected region R prove that the line integral $\int_a^b f(z)\ dz$ is independent of the path joining any two points a and b in R. 5+5=10

b. State Green's theorem in the plane. Verify Green's theorem for the integral $\oint_C (2y - x^2)dx + (x + y^2)\ dy$ where C is the closed curve between the parabolas $y = x^2$ and $x = y^2$.

= = *** = =