REV-01
MSM/25/30

2024/05

SET A

M.Sc. MATHEMATICS
SECOND SEMESTER
COMPLEX ANALYSIS
MSM – 201

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. $f(z)=\overline{z}$ is
   a. continuous for every z, not differentiable for any z
   b. continuous for some values of z, differentiable for every z
   c. discontinuous for every z, differentiable for every z
   d. neither continuous nor differentiable

2. Functions satisfying Laplace's equation are known as
   a. regular
   b. homomorphic
   c. harmonic
   d. conjugate

3. An analytic function with constant modulus is
   a. variable
   b. constant
   c. zero
   d. Doesnot exist

4. Function $f(z)=xy+iy$ is
   a. everywhere continuous and analytic
   b. everywhere continuous but not analytic
   c. discontinuous but analytic everywhere
   d. neither continuous nor analytic

5. If only one value of function corresponds to each value of complex number is called
   a. Single valued
   b. Multiple valued
   c. Both a and b
   d. None

6. If $f(z)$ is continuous in a closed region, then the function is
   a. bounded
   b. unbounded
   c. Doesnot exist
   d. None

7. The value of $\lim_{z\to\infty}\left(e^{-z}+\frac{1}{z}\right)$ is
   a. 1
   b. 0
   c. -1
   d. None

8. The value of $\oint_C \frac{dz}{z-a}$ where C is any simple closed curve and $z = a$ is

a. 0
b. 1
c. $2\pi i$
d. None

9. The singularity at $z = 2$ of $f(z) = e^{\frac{1}{z-2}}$ is called

a. Pole
b. Essential singularity
c. Removable singularity
d. None

10. A continuous arc without multiple points is called a

a. Jordan curve
b. Continuous arc
c. Contour
d. Rectifiable arc

11. The value of $\int_C \frac{dz}{z}$ where C is the circle with centre at the origin and radius r is

a. $\log r$
b. πi
c. $2\pi i$
d. $\frac{\pi i}{2}$

12. A continuous function $f(z)$ over a continuous rectifiable curve C is

a. Differentiable
b. Integrable
c. Meromorphic
d. None

13. Every analytic function in a simply-connected domain

a. Possesses a definite integral
b. Possesses an indefinite integral
c. Doesnot possesses an indefinite integral
d. None

14. Polynomial of degree n has a pole of order n at

a. Zero
b. Infinity
c. Curve C
d. Anywhere

15. A function whose only singularities in the entire complex plane are poles, is called

a. Analytic
b. Homomorphic
c. Meromorphic
d. Regular

16. Function e^z has at $z = \infty$

a. An isolated singularity
b. A pole
c. An infinite point
d. An isolated essential singularity

17. Residue of $\frac{z^3}{z^2-1}$ at $z=\infty$ is

a. 1
b. -1
c. 0
d. ∞

18. The value of $\lim_{z \to i} \frac{z^2+1}{z^6+1}$ is

a. $\frac{1}{3}$
b. $\frac{1}{2}$
c. $\frac{1}{5}$
d. $\frac{1}{7}$

19. If $u=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic then its harmonic conjugate is

a. $\log\left(\frac{y}{x}\right)+c$
b. $\log\left(\frac{x}{y}\right)+c$
c. $\tan^{-1}\left(\frac{y}{x}\right)+c$
d. $\tan^{-1}\left(\frac{x}{y}\right)+c$

20. If C be the circle of $|z|=1$, then the value of $\int_C \frac{z\,dz}{z-2}$

a. 1
b. 2
c. $2\pi i$
d. 0

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. Using Cauchy integral formula calculate the integrals: **3+4+3 =10**

a. $\int_C \frac{\cosh(\pi z)\,dz}{z(z^2+1)}$ where C is circle $|z| = 2$.

b. $\int_C \frac{e^{iz}\,dz}{(z-\pi i)}$ where C is the ellipse $|z-2|+|z+2| = 6$.

c. $\oint_C \frac{e^{2z}\,dz}{(z+1)^4}$ where C is the circle $|z| = 3$.

2. Verify Green's theorem in the plane for $\oint_C x^2\,y dx + (y^3 - xy^2)\,dy$ **10**

where C is the boundary of the region enclosed by the circles $x^2+y^2 = 4, x^2+y^2 = 16$.

3. **a.** Show that $u = \frac{1}{2}\log(x^2+y^2)$ is a harmonic function and find its harmonic conjugate. Also find the analytic function in terms of z. **6+4=10**

b. Show that an analytic function with constant modulus is constant.

4. Prove that if $f(z)$ is continuous in a simply connected region R and $\oint_C f(z)dz = 0$ around every simple closed curve C in R, and then $f(z)$ is analytic in R. **10**

5. State and prove Taylor's theorem. 10

6. **a.** Evaluate $\oint_C \frac{e^{zt}dz}{z^2(z^2+2z+2)}$ around the circle C with $|z| = 3$. 5+5=10

b. Find the bilinear transformation which maps $z = 1, i, -1$ onto $w = i, 0, -i$.

7. What kind of singularities the following functions have 2×5=10

a. $f(z) = \frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$

b. $f(z) = \frac{\cot \pi z}{(z-a)^2}$ at $z = 0$ and $z = \infty$

c. $f(z) = \frac{1-e^z}{1+e^z}$ at $z = \infty$

d. $f(z) = \tan\frac{1}{z}$ at $z = 0$

e. $f(z) = \cos ec\frac{1}{z}$ at $z = 0$

8. **a.** Write the statement of Milne-Thomson method. Find the regular function $f(z) = u + iv$ where 2+4+4 =10

$$u = e^{-x}\left\{(x^2 - y^2)\cos y + 2xy\sin y\right\}.$$

b. Evaluate $\lim_{z \to 2e^{\frac{\pi i}{3}}} \frac{z^3+8}{z^4+4z^2+16}$.

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