2024/05

SET A

M.Sc. MATHEMATICS
FOURTH SEMESTER
PARTIAL DIFFERENTIAL EQUATION II
MSM - 402

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. A second order PDE in two variables is hyperbolic if
   a. $S^2 - 4RT = 0$
   b. $S^2 - 4RT < 0$
   c. $S^2 - 4RT > 0$
   d. none of the above

2. Two-dimensional wave equation is
   a. $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), c^2 = \frac{T}{\rho}$
   b. $\frac{\partial^2 u}{\partial t^2} = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$
   c. $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), c^2 = \frac{T}{\rho}$
   d. $\frac{\partial u}{\partial t} = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

3. The equation of motion of one-dimensional wave equation using Newton's second law is
   a. $T_2 \sin\beta - T_1 \sin\alpha = (m\partial s)\frac{\partial^2 u}{\partial t^2}$
   b. $T_2 \sin\beta - T_1 \sin\alpha = (m\partial s)\frac{\partial u}{\partial t}$
   c. $T_2 \sin\beta - T_1 \sin\alpha = (\rho\partial s)\frac{\partial u}{\partial t}$
   d. $T_2 \sin\beta - T_1 \sin\alpha = (\rho\partial s)\frac{\partial^2 u}{\partial t^2}$

4. What is $\lambda = ?$
   For the PDE $U_{xx} + U_{yy} = U_{zz}$?
   a. $\lambda = 1,1,1$
   b. $\lambda = 1,1,-1$
   c. $\lambda = -1,-1,-1$
   d. None of the above

5. The Laplace Equation in three dimension is
   a. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u$ is heat flow
   b. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0, u$ is heat flow

c. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0, u$ is wave flow

d. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, u$ is wave flow

6. The Charpit's auxillary equation for the equation $p^2-q^2=x-y$ is

a. $\frac{dx}{-2p}=\frac{dy}{-2q}=\frac{dz}{-p(2p)-q(2q)}=\frac{dp}{-1}=\frac{dq}{1}$

b. $\frac{dx}{2p}=\frac{dy}{2q}=\frac{dz}{-p(2p)-q(2q)}=\frac{dp}{-1}=\frac{dq}{1}$

c. $\frac{dx}{-2p}=\frac{dy}{2q}=\frac{dz}{-p(2p)-q(2q)}=\frac{dp}{1}=\frac{dq}{1}$

d. $\frac{dx}{-2p}=\frac{dy}{2q}=\frac{dz}{-p(2p)-q(2q)}=\frac{dp}{-1}=\frac{dq}{1}$

7. The Differential equation $(p-q)(z-px-qy)=1$ is

a. a first order non linear PDE
b. a first order Linear PDE
c. a second order non linear PDE
d. a second order linear PDE

8. The Clairaut's equation in PDE is

a. $z = px + qy$
b. $y = px + f(p)$
c. $z = px + qy + f(p,q)$
d. $y = x + pq$

9. For derivation of two-dimensional wave equation,we consider

a. A rod
b. A string
c. A membrane
d. None of the above

10. The Partial Differential equation $xyr-(x^2-y^2)s-xyt+py-qx=2(x^2-y^2)$ is

a. hyperbolic
b. elliptic
c. parabolic
d. Parabolic and hyperbolic

11. If the particular integral of the partial differential equation $(D^2+2DD'+D'^2)z=\cos(x+2y)$ is $k\cos(x+2y)$, then value of k is

a. $-\frac{1}{9}$
b. $\frac{1}{9}$
c. $\frac{1}{8}$
d. $-\frac{1}{8}$

12. The complete solution of the PDE $q(p-\cos x)=\cos y$ is

a. $z=ax-\sin x-\frac{\sin y}{a}+b$
b. $z=ax+\sin x-\frac{\sin y}{a}+b$
c. $z=ax-\sin x+\frac{\sin y}{a}+b$
d. $z=ax+\sin x+\frac{\sin y}{a}+b$

13. Which of the following equation is elliptic
   a. Laplace equation
   b. Heat equation
   c. Wave equation
   d. $u_{xx} + 2u_{xy} - 4u_{yy} = 0$

14. Choose the region in which the following PDE is hyperbolic
   a. $xy \neq 1$
   b. $xy \neq 0$
   c. $xy > 1$
   d. $xy > 0$

15. Which of the following is not true about the solution of a cauchy problem
   a. Solution does not exist
   b. Solution unique
   c. Infinitely many solutions
   d. None

16. The PDE $u_{xx} + u_{yy} = 0$ is
   a. Hyperbolic
   b. Parabolic
   c. Elliptic
   d. None

17. Which equation represents a wave equation
   a. $u_{xx} + u_{tt} = 0$
   b. $u_{xx} - u_{tt} = 0$
   c. $u_{xx} + u_{t} = 0$
   d. $u_{xx} - u_{t} = 0$

18. Which one is harmonic?
   a. $u(x, y) = \sin x + \cos y$
   b. $u(x, y) = \sin xy$
   c. $u(x, y) = \cos xy$
   d. $u(x, y) = x + y$

19. Heat equation is also called a
   a. Elliptic
   b. Hyperbolic
   c. Parabolic
   d. None

20. Separation of variables method is also called
   a. Fourier method
   b. Laplace method
   c. By parts method
   d. None

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. A homogeneous rod of conducting material of length 'a' has its ends kept at zero temperature. The temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature function $u(x,t)$. **10**

2. Find the complete integral of following partial differential equation **5+5=10**
   a. $pxy + pq + qy = yz$
   b. $2zx - px^2 - 2qxy + pq = 0$

3. Obtain most suitable solution of the wave equation **10**

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, \quad \text{where } c^2 = \frac{T}{m}$$

Subject to the condition boundary condition

And initial condition $u(0,t) = u(l,t) = 0$

and $u(x,o) = f(x)$, $\dfrac{\partial u}{\partial t} = 0$ at $t = 0$

where l is a length of string or wire.

4. Write two difference between Laplace Equation in plane polar coordinate and Cylindrical coordinate.Prove that **2+8=10**

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \times \left(\frac{\partial v}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 v}{\partial \theta^2}\right) + \frac{\partial^2 v}{\partial z^2} = 0$$

5. What are the first three steps to canonical form. Reduce the following differential equation to canonical form **,3+7=10**

$$y^2 \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} = 0$$

6. Find the particular solution of the PDE $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c}\frac{\partial u}{\partial t}$, subject to the boundary conditions $u(0,t) = u(a,t) = 0 \ \forall t$ and the initial condition is $u(x,0) = f(x), 0 < x < a$. **10**

7. Find the solution of $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ satisfying **10**

i. u is bounded as $t \to \infty$

ii. $u_x(0,t) = u_x(a,t) = 0 \ \forall t$

iii. $u(x,0) = x(a-x), 0 < x < a$

8. Find the temperature distribution inside a square plate of side 'a' having boundary conditions **10**

$u(0,y,t) = u(a,y,t) = u(x,o,t) = u(x,a,t) = 0$ and the initial condition is

$$u(x,y,0) = \cos\left\{\frac{\pi(x-y)}{a}\right\} - \cos\left\{\frac{\pi(x+y)}{a}\right\}.$$

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