REV-01
MSM/01/05

2024/05

**SET A**

# M.Sc. MATHEMATICS
## FOURTH SEMESTER
## GRAPH THEORY
## MSM – 401 [REPEAT]
[USE OMR FOR OBJECTIVE PART]

**Duration: 3 hrs.** **Full Marks: 70**

**Objective**

**Time: 30 min.** **Marks: 20**

***Choose the correct answer from the following:*** ***1X20=20***

1. Two graph are said to be isomorphic if
   a. Both have same numbers of vertices
   b. Both have same numbers of vertices, edges & with same connectivity
   c. Both have the same numbers of edges
   d. None of the above

2. What is the number of edges present in complete graph $K_n$ having n vertices?
   a. $\frac{n(n+1)}{2}$
   b. $\frac{n(n-1)}{2}$
   c. $n^2$
   d. None of these

3. What is the maximum number of edges in a bipartite graph having 12 vertices.
   a. 24
   b. 38
   c. 36
   d. 32

4. A Graph contains m edges can be decomposed in
   a. $2^m - 1$ ways
   b. $2^{m-1} - 1$ ways
   c. $2^m$ ways
   d. None

5. The number of Hamiltonian Circuits in a complete graph of n vertices
   a. $\frac{(n-1)!}{2}$
   b. $\frac{n!}{2}$
   c. $\frac{(n-2)!}{2}$
   d. None of these

6. What is the determinant of the adjacency matrix of $C_4$
   a. 1
   b. $-1$
   c. 0
   d. None of these

7. Conditions of planarity for n numbers of vertices and e numbers of edges
   a. $e < 2n - 6$
   b. $e < 2n - 6$
   c. $e \leq 3n - 6$
   d. $e = 2n - 6$

8. Number of spanning tree of a graph of n vertices is
   a. 0 b. 2
   c. $2^n$ d. None

9. No of cycles in a connected planar graph having 6 vertices, 7 edges is ____ _____regions.
   a. 1 b. 3
   c. 5 d. 7

10. A graph is self- complementary if it is isomorphic to its complement. For all self-complementary graphs on $n$ vertices,$n$ is
   a. Amultipleof4 b. Even
   c. Odd d. congruentto0$mod4$,or1$mod4$

11. A vertex which is not adjacent to every other vertex is called__vertex.
   a. Isolated b. Pendant
   c. Incident d. Simple

12. Let $n_1$ and $n_2$ be the number of edges of graphs $G_1$and$G_2$ respectively, then the number of edges of $G1 \cup G2$ if $G_1$and$G_2$ are connected graphs
   a. $n_1 + n_2$ b. $n_1 + n_2 - 1$
   c. $n_1$ d. $n_2$

13. For a given graph G having v vertices and e edges which is connected and has no cycles, which of the following statements is true?
   a. $v = e$ b. $v = e + 1$
   c. $v + 1 = e$ d. $v = e - 1$

14. What is the radius of the Petersengraph
   a. 2 b. 3
   c. 4 d. None of these

15. Total number of trees with 5 vertices
   a. 2 b. 3
   c. 4 d. 5

16. Which of the following statement is/are TRUE?
   P: A cycle is walks with end vertices are same.
   Q: A cycle is a path with end vertices is same.
   a. P only b. Q only
   c. Both P and Q d. Neither P and Q

17. A graph is called a_____if it is a connected a cyclic graph
   a. Cyclic graph b. Regular graph
   c. Tree d. Trivial graph

18. The graph in which, there is a closed trail which includes every edge of the graph is known as?
    a. Hamiltonian graph
    b. Euler graph
    c. Directed graph
    d. Planar graph

19. The number of elements in the adjacency matrix of a graph having 7 vertices is _____
    a. 7
    b. 14
    c. 36
    d. 49

20. G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be
    a. Regular
    b. Complete
    c. Hamiltonian
    d. Euler

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## Descriptive

**Time : 2 hrs. 30 mins.** **Marks : 50**

***[ Answer question no.1 & any four (4) from the rest ]***

1. With diagrams define of the followings **2×5=10**
    a. General Graph
    b. Simple graph
    c. Regular graph
    d. Complete graph
    e. Isomorphic Graph

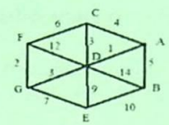
2. **5+5=10**
    a. If m and M denote the minimum and maximum degrees of the vertices of a graph with $n_V$ vertices and $n_E$edges, Show that $m \leq \frac{2n_E}{n_V} \leq M$

    b. If the number of vertices and number of edges of two graphs$G_1$ $(p_1, q_1)$ and $G_2$ $(p_2, q_2)$then define with example by the rule of $G_1 \cap G_2$, $G_1 \cup G_2$ and $G_1 \oplus G_2$

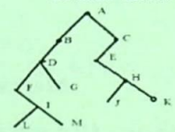
3. **a.** If A is the adjacency matrix with $V = \{v_1, v_2, v_3, \ldots\ldots\ldots v_i\}$, then prove that for any $n \geq 1$ the $(i,j)th$ entry of $A^n$ is the number of $v_i - v_j$ walks of length n in G. **5+5=10**

**b.** Prove that G with P vertices and $\delta \geq \frac{p-1}{2}$ is connected.

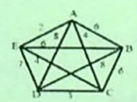
4. **a.** Find a minimum spanning tree for the weighted graph by using prim algorithm of the following graph **5+5=10**




**b.** List the order in which the vertices of the tree given below are processed using pre-order, in-order and post-order traversal.




5. a. Find the minimum spanning tree for the weighted graph as given below by using Kruskal algorithm **5+5=10**




b. Find the value of the prefix expression

$$+-\uparrow 32 \uparrow 23/8 - 42$$

6. a. Represent the expression $((a-c)\times d))/(a+(b-d))$ as a binary tree and write prefix and postfix forms of the expression. **5+5=10**

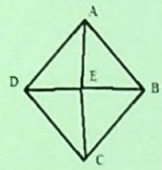
   b. Find the value of the postfix expression
   $$72\ 3\ +\ 232\ +\ -13\ -*/.$$

7. a. If G is a graph with $p \geq 3$ vertices and $\delta \geq \frac{p}{2}$, then prove that G is Hamiltonian. **5+5=10**

   b. If G is a connected planer graph having V, E, and F as the sets of vertices , edges and faces respectively, then prove that

   c. $|V| - |E| + |F| = 2$

8. a. Find the chromatic polynomial of the below ABCDE graph **5+5=10**




   b. Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.

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