M.Sc. PHYSICS
THIRD SEMESTER
MATHEMATICAL PHYSICS-II
MPH-301

Duration: 3 Hrs. **Marks: 70**

PART : A (OBJECTIVE) = 20
PART : B (DESCRIPTIVE) = 50

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. Express the Bessel differential equation. Obtain the solution of Bessel's function. Draw the graphs of the two functions $J_0(x)$ and $J_1(x)$. (1+7+2=10)

2. (a) Express the following function in Fourier- Legendre expansion. (4+3+3=10)

$$f(x) = \begin{matrix} 0 & -1 \le x \le 0 \\ 1 & 0 \le x \le 1 \end{matrix}$$

(b) (i) Prove that:

$$J_{1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \operatorname{Sin} x$$

(ii) Prove that: $nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-2}$

3. Obtain the path integral equation for a free particle. (10)

4. (a) State the Orthogonality Theorem and prove it for $d \neq d'$. (6+4=10)
(b) What is a unitary group? Show the Unitary representation of a group D_3 and hence show the reducible representation of an equilateral triangle.

5. (a) Show that there is homomorphism between SU(2) and SO(3)generators. (5+5=10)
(b) Prove the orthogonality of Bessel function.

6. (a) Write a brief note on Homomorphism and Isomorphism of a group with suitable example. (7+3=10)
(b) Show that a Group of non-zero complex numbers is a homomorphism.

7. (a) Prove that: (3+7=10)

(i)

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$$

Or

(ii) Prove that

$$\int_{-1}^{+1} [P_n(x)]dx = \frac{2}{2n+1}$$

(b) Obtain the integral form of Linear harmonic oscillator equation by transformation of its differential form into homogenous Fredholm Integral equation of second kind.

8. (a) If $u(x) = e^{-x^2}$ is a solution of the Volterra integral equation (3+3+4=10)

$$u(x) = 1 - \alpha \int_0^x t\, u(t)\, dt$$

Find α?

(b) Check if $u(x) = x + e^x$ is a solution of the Fredholm integral equation

$$u''(x) = e^x - \frac{4}{3}x + \int_0^1 xt\, u(t)\, dt; \quad u(0) = 1, \quad u'(0) = 2$$

(c) Prove that $J_n(x)$ is a coefficient of z^n in the expansion of $e^{x/2(z-\frac{1}{z})}$.

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[PART-A : Objective]

Choose the correct answer from the following: **1×20=20**

1. The unitary matrix are precisely those matrices which preserve the Hermitian inner product:
   a. $< v, w > = < Uw, vU >$
   b. $< v, w > = < Uv, Uw >$
   c. $< v, w > = < Uu, wU >$
   d. $< v, w > = < Uw, Uw >$

2. A function defined by $f(0)= f(2\pi n)=1$, where $n= 1,2, \ldots..n$, then the function is a:
   a. Isomorphism
   b. Endomorphism
   c. Homomorphism
   d. None of the above

3. If $ab= ba$ for all a, b of a group G, the multiplication is commutative, and the group is called:
   a. Conjugate subgroup
   b. An abstract group
   c. Cyclic group
   d. An abelian group

4. The orthogonality condition of Hermite polynomial for $(m \neq n)$ is given by:
   a. $\int_{-\infty}^{+\infty} H_m(x) H_n(x)\, e^{-x^2}\, dx = 0$
   b. $\int_{-\infty}^{+\infty} \varphi_m(x)\, \varphi_n(x)\, dx - 0$
   c. $\int_{-\infty}^{+\infty} \varphi_m(x)\, \varphi_n(x) e^{-x^2}\, dx = 0$
   d. $\int_{-\infty}^{+\infty} H_m(x) H_n(x)\, e^{x^2}\, dx - 0$

5. Choose the incorrect option from the following:
   a. A cyclic group is also abelian.
   b. An abelian group is also cyclic.
   c. If 'a' group possesses an element $a, a^2, a^3, \ldots.$, then the group is cyclic.
   d. All of the above.

6. The generating function of Laguerre polynomial L_n is:
   a. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n(n-1)!} t^n$
   b. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n(n+1)!} t^n$
   c. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n!} t^n$
   d. $\sum_{n=0}^{\infty} \frac{L_n(x)}{(n-1)!} t^n$

7. The recurrence formula for Bessel function of the form $\frac{d}{dx}(x^n J_n)$ is equal to:
   a. $x^n J_n$
   b. $x^n J_{n+1}$
   c. $x^n J_{n+2}$
   d. $x^n J_{n-1}$

8. The equation $g(x) = f(x) + \int_a^b dt\, K(x, t) f(t)$ is a:
   a. Volterra equation of first kind.
   b. Volterra equation of second kind.
   c. Fredholm equation of first kind.
   d. Fredholm equation of second kind.

9. If the unknown appears only under the integral sign, we label it as:
   a. Volterra equation of first kind.
   b. Volterra equation of second kind.
   c. Fredholm equation of first kind.
   d. Fredholm equation of second kind.

10. $iS = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
    means
    $S = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is a Pauli matrix given by:
    a. σ_x
    b. σ_y
    c. σ_z
    d. None of the above

11. The eight SU(3) generators can be represented in terms of zero-trace Hermitian matrices with
    $S_i = \frac{1}{2} \lambda_i$
    The λ_i are known as:
    a. Diagonal matrices
    b. Unitary matrices
    c. Gell- Mann matrices
    d. All of the above

12. If the Kernel $K'(x,t) = K(t,x)$, it is:
    a. asymmetric
    b. continuous
    c. discontinuous
    d. symmetric

13. The quantity $e^{-\frac{iH(t-t_0)/\hbar}{}}|x\rangle$ is the:
    a. Phase of the particle.
    b. State of the object at time 't'.
    c. Probability amplitude of the particle.
    d. None of the above.

14. The operator $dx_1\,|x_1\rangle\langle x_1|$ is called:
    a. An ordinary propagator
    b. A propagator
    c. An identity operator
    d. All of the above

15. A relation between two vector spaces V and W is a map $T : V \to W$ such that the following holds $T(\alpha \mathbf{v}) = \alpha T(\mathbf{v})$, for any scalar α is a:
    a. Linear transformation
    b. Non-Linear transformation
    c. One- to- one transformation
    d. None of the above

16. If vectors are linearly independent and every vector in the vector space is a linear combination of this set, then a set of elements in a vector space V is called:
    a. Basis
    b. Vector coordinates
    c. Set of basis vectors
    d. All of the above

17. The transformations that are consistent with the symmetry of space-time form a group known as:
    a. Homogeneous Lorentz group
    b. Ordinary Lorentz group
    c. Poincaré Group
    d. None of the above

18. The eight baryons which are symmetrically related are:
    a. Irreducible representation of a group SU(3)
    b. Reducible representation of a group SU(3)
    c. Irreducible representation of a group SU(2)
    d. Reducible representation of a group SU(2)

19. All angular momentum multiplets define representations of:
    a. SU(3) group
    b. SU(2) group
    c. SO(2) group
    d. SO(3) group

20. The number of generators of a Lie group is equal to the:
    a. Basis of the group
    b. Parameter of the group
    c. Order of the group
    d. None of the above

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

[PART (A) : OBJECTIVE]

Duration : 20 Minutes

Serial no. of the main Answer sheet

Course : .....

Semester : ..... **Roll No :** .....

Enrollment No : ..... **Course code :** .....

Course Title : .....

Session : 2017-18 **Date :** .....

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Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- Students shall tick (✓) the correct answer.
- No marks shall be given for overwrite / erasing.
- Students have to submit the Objective Part (Part-A) to the invigilator just after completion of the allotted time from the starting of examination.

Full Marks	Marks Obtained
20	

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*