REV-01
BSC/05/10

2024/06

SET A

B.Sc. MATHEMATICS
FOURTH SEMESTER
CALCULUS
BSM – 741 OLD COURSE [REPEAT]
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. The domain of the function $\frac{1}{\sqrt{x}}$ is
   a. $[0, \infty)$ b. $(0, \infty)$
   c. $[\infty, \infty)$ d. None of the above

2. The sides of an equilateral triangle increases at the rate of $\frac{1}{\sqrt{3}} cm/sec$. What will be the rate of increase in area of the triangle when the length of a side is 10 c.m?
   a. $50cm^2/sec$ b. $15cm^2/sec$
   c. $5cm^2/sec$ d. None of the above

3. $$\int \frac{\cos(logx)\,dx}{x} = ?$$
   a. $\sin(logx) + c$ b. $\sec(logx) + c$
   c. $\cos(logx) + c$ d. None of the above

4. $\lim_{n\to\infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!} = ?$
   a. 1 b. -1
   c. 0 d. 2

5. Find the value of $\int \frac{1}{e^x+1} dx$
   a. $\log(1 + e^{-x})$ b. $-\log(1 + e^{x})$
   c. $-\log(1 + e^{-x})$ d. None of the above

6. Give the equivalents of the inequality $-3 \le x \le 7$
   a. $|x - 2| \le 5$ b. $|x - 2| \le -5$
   c. $|x + 2| \le 5$ d. None

7. What is the value of $\int secx\,dx$
   a. $\log(cosx + tanx)$ b. $\log(cosecx + tanx)$
   c. $\log(secx + tanx)$ d. None of the above

8. Find the value of $\int \frac{dx}{\sqrt{x^2+a^2}}$
   a. $\log|x + \sqrt{x^2 + a^2}|$ b. $\log|x - \sqrt{x^2 + a^2}|$
   c. $\frac{1}{a}\tan^{-1}\frac{x}{a}$ d. None of these

9. Evaluate $\lim_{x\to 0}\frac{1-cosx}{x^2}$

a. $\frac{1}{7}$ b. $\frac{1}{4}$

c. $\frac{1}{3}$ d. $\frac{1}{2}$

10. The value of $\lim_{x\to 0}\frac{e^x-1}{x}$ is

a. 0 b. -1

c. ∞ d. 1

11. Find the value of $\int logx\,dx$

a. $xlogx+x$ b. $xlogx$

c. $xlogx-x$ d. None of the above

12. Find $\frac{d^2y}{dx^2}$ if $y=\frac{logx}{x}$

a. $\frac{2logx-3}{x^3}$ b. $\frac{2logx-2}{x^3}$

c. $\frac{2logx-3}{x^2}$ d. None

13. What is the second derivative of $y=\frac{logx}{x}$

a. $\frac{(2logx-3)}{x^3}$ b. $\frac{(2logx-3)}{x^2}$

c. $\frac{(2logx-5)}{x^3}$ d. None

14. If $y=be^x+ce^{2x}$ then $y_2-3y_1=?$

a. $-2y$ b. $5y$

c. $2y$ d. none

15. $\lim_{x\to 0}\frac{1}{x^2}=?$

a. 0 b. 1

c. ∞ d. None of the above

16. Derivative of $\log sinx$ is

a. cotx b. tanx

c. secx d. None of the above

17. $If\ siny=x\sin(a+y),\ then\ \frac{dy}{dx}=?$

a. $\frac{\sin^2(a+y)}{sina}$ b. $\frac{\cos(a+y)}{sina}$

c. $\frac{\sin^3(a+y)}{sina}$ d. None of the above

18. Find the value of $\int \frac{dx}{\sqrt{a^2-x^2}}$

a. $\tan^{-1}\frac{x}{a}|x| < |a|$ b. $\sin^{-1}\frac{x}{a}|x| < |a|$

c. $\cos^{-1}\frac{x}{a}|x| < |a|$ d. None of the above

19. Derivative of $\cosh x$ is

a. $-\sinh x$ b. $\operatorname{sech} x$

c. $\sinh x$ d. None of the above

20. Evaluate $\int_0^{\pi/2} \frac{dx}{1+\cot x}$

a. $\frac{\pi}{2}$ b. π

c. $\frac{\pi}{4}$ d. None of these

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. **a.** Evaluate $\int \frac{dx}{3-5\sin^2 x}$. **5+5=10**

b. If $x = 2cos\theta - cos2\theta, y = 2sin\theta - sin2\theta,$ then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \pi/2$.

2. **a.** Find $\frac{dy}{dx}$ if $y = tan^{-1}\frac{cos2x}{1+sin2x}$ **4+3+3 =10**

b. Define continuity and discontinuity with examples. Show that $f(x) = 3x^2 + 2x - 1$ is continuous at 2.

3. **a.** If $y = x^3 log\frac{1}{x}$ then show that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$ **4+6 =10**

b. Find $\frac{dy}{dx}$If (i) $y = [x^{tanx} + (sinx)^{cosx}]$ and (ii) $x^y . y^x = 1$

4. **a.** (i)Find the derivative of $cosx$ using limit. **3+3+4**
(ii) If $x = e^{-t^2}$ *and* $y = tan^{-1}(2t+1)$, *find* $\frac{dy}{dx}$. **=10**

b. Evaluate $\lim_{x\to 0} \frac{\log(5+x)-\log(5-x)}{x}$

5. **a.** Evaluate $\int \frac{dx}{5-13\,sinx}$ **5+5=10**

b. Find by the Newton's method (upto third approximation), the real root of the equation $x^2 - 3 = 0$

6. **a.** Evaluate the following integrals **3+3+4**
(i) $\int \tan^5 x\ dx$ (ii) $\int_0^1 x\ tan^{-1}x\ dx$ **=10**

b. Evaluate By the method of partial fraction $\int \frac{(x-1)dx}{(x-2)(x-3)}$

7. **a.** Evaluate $\int \sqrt{\frac{a+x}{a-x}}dx$ **5+5=10**

b. Evaluate $\int \log(x + \sqrt{x^2+a^2})dx$

8. a. (i) At a certain instant the volume of a sphere increases at the rate of $36cm^3/sec$ and its surface area increases at the rate of $12\ cm^2/sec$. Find the rate of change of its radius at that instant. **3+3+4**
(ii) If $y = x \log\frac{x}{a+bx}$ *show that* $x^3y_2 = (xy_1 - y)^2$ **=10**

b. Evaluate $\int x^5 \tan^{-1} x^3\, dx$

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