REV-01
BSM/02/05

2024/07

SET A

B.Sc. MATHEMATICS
THIRD SEMESTER
CLASSICAL ALGEBRA & TRIGONOMETRY
BSM - 731[SPECIAL REPEAT]

[USE OMR FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

Objective

Time : 30 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. If α, β, γ are roots of the equation $x^3 + qx + r = 0, then \sum \alpha\beta = ?$
   a. q b. $-r$
   c. r d. none

2. The condition $AM = GM$ holds when the quantities are
   a. equal b. unequal
   c. Hold for any numbers d. none

3. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then what is the value of $\sum \alpha^2\beta$
   a. $-3r$ b. $3r$
   c. $2r$ d. none

4. In which case a system of equations has no solution?
   a. (adj A).B$\neq$ 0 b. (adj A).B=0
   c. (adj A). B < 0 d. none

5. The value of i^i is
   a. $e^{-(4n+1)\frac{\pi}{2}}$ b. $e^{-(4n+1)\frac{\pi}{3}}$
   c. $e^{-(4n+1)\frac{\pi}{8}}$ d. none

6. The value of $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 = ?$, where ω is the cube root of unity
   a. 1 b. 2
   c. -1 d. 0

7. If the sum of two roots of the equation $x^3 + a_1x^2 + a_2x + a_3 = 0$ *is zero, then*
   a. $a_1a_2 = -a_3$ b. $a_1a_2 = a_3$
   c. $a_2 = a_3$ d. none

8. If a, b, c are positive numbers then, which of the following is true?
   a. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq 3$ b. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq 9$
   c. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3$ d. none

9. Find the determinant of $M=\begin{bmatrix}1 & bc & a(b+c)\\ 1 & ca & b(c+a)\\ 1 & ab & c(a+b)\end{bmatrix}$

a. 1 b. 0

c. $bc+ca+ab$ d. none

10. Find x, y, z *and* t *which satisfy the matrix equation*

$$\begin{bmatrix}x-y & 2x+z\\ 2x-y & 2z+t\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 0 & 13\end{bmatrix}$$

a. $x=1, y=2, z=3, t=-1$ b. $x=1, y=-2, z=3, t=-1$

c. $x=1, y=2, z=3, t=7$ d. none

11. If z is a non-zero complex number, then $z\bar{z}$ *is*

a. Purely real b. Purely imaginary

c. zero d. none

12. Which of the following is not a symmetric function?

a. $x^2+y^2+z^2$ b. $x+y$

c. $x-y$ d. xy

13. If α, β are roots of $x^2-2x+4=0$, then what is the value of α^n?

a. $2^n\left(cos\frac{n\pi}{3}+isin\frac{n\pi}{3}\right)$ b. $2^n\left(cos\frac{n\pi}{6}+isin\frac{n\pi}{6}\right)$

c. $2^{n+1}\left(cos\frac{n\pi}{3}+isin\frac{n\pi}{3}\right)$ d. none

14. Which of the following will not hold for any positive real numbers belongs to R, if $a>b$,then

a. $a+c>b+c$ b. $a-c>b-c$

c. $ac<bc$ d. none

15. Which inequality is known as Cauchy- Schwartz's inequality?

a. $(\sum_{i=1}^n a_i b_i)\geq(\sum_{i=1}^n a_i)\ (\sum_{i=1}^n b_i)$ b. $(\sum_{i=1}^n a_i b_i)^2\leq(\sum_{i=1}^n a_i^2)\ (\sum_{i=1}^n b_i^2)$

c. $(\sum_{i=1}^n a_i^2 b_i^2)\leq(\sum_{i=1}^n a_i{}^2)(\sum_{i=1}^n b_i^2)$ d. none

16. Which is the correct expression for Gregory series?

a. $\theta=tan\theta+\frac{\tan^3\theta}{3}+\frac{\tan^5\theta}{5}....$ b. $\theta=tan\theta-\frac{\tan^3\theta}{3}-\frac{\tan^5\theta}{5}....$

c. $\theta=tan\theta-\frac{\tan^3\theta}{3}+\frac{\tan^5\theta}{5}....$ d. none

17. Evaluate the value of $\frac{(cos\theta-isin\theta)^{10}}{(cos\theta+isin\theta)^{12}}$

a. $cos22\theta-isin22\theta$ b. $cos22\theta+isin22\theta$

c. $cos12\theta-isin12\theta$ d. none

18. The value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is

a. $(a-b)(b-c)(c-a)$ b. $-(a-b)(b-c)(c-a)$

c. $(a-b)(b-c)$ d. none

19. If $x = 5+2i, y = 5-2i, then\ x^2+y^2+xy =?$

a. -71 b. 71

c. 17 d. none

20. Find the cofactor of 6 of the determinant $\begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$

a. -4 b. 5

c. 4 d. none

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Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Define polynomial and fundamental theorem of Arithmetic. Find the conditions under which the roots of the equation $x^3+px^2+qx+r=0$ are in (i) AP (ii) GP **2+8=10**

2. **a.** State Cauchy Schwartz Inequality. If $a^2+b^2+c^2=1$ then show that $-\frac{1}{2} \le ab+bc+ca \le 1$. **1+4+5 =10**

   b. If a, b, c are positive and distinct show that $\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b} > 6$.

3. **a.** State and prove De Moivre's Theorem. **8+2=10**
   b. Give the exponential expansion of Sine and Cosine

4. **a.** Define upper triangular, lower triangular and diagonal matrix with example. **6+4=10**

b. Find the rank of the matrix A= $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

5. **a.** State and deduce Gregory's series . **4+3+3 =10**

b. (i) Prove that $sin\left(i\ log\frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2+b^2}$

(ii) Prove that $\log(1 + i\ tan\theta) = \log sec\theta + i\theta$

6. **a.** Solve the system of equation by matrix method **6+4=10**

$$x + 2y + 3z = 4,$$
$$4x + 4y + 9z = 6$$
$$x + y + z = 3.$$

b. If A= $\begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ find AB' *and* BA'.

7. **a.** Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ given that the roots are in geometric progression. **5+5=10**

b. If $\alpha, \beta$and γ are the roots of the equation $x^3 + px + q = 0$ then find the vales of $\sum \alpha^2$ and $\sum \frac{1}{\alpha+\beta}$.

8. **a.** If $\tan(x + iy) = u + iv$ then prove that $u^2 + v^2 + 2ucot2x = 1$. **4+3+3 =10**

b. (i) Prove that $(1 + cos\theta + isin\theta)^n = 2^n \cos^n\frac{\theta}{2}(cos\frac{n\theta}{2} + isin\frac{n\theta}{2})$.

(ii) Determine the equation in complex form of a circle with centre
(-3, 4) and radius 2.

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