2024/06

M.SC. MATHEMATICS
SECOND SEMESTER
PARTIAL DIFFERENTIAL EQUATION I
MSM – 205

SET A

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(Objective)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1×10=10***

1. Monge's method is used to solve a Partial Differential Equation of
   a. nth order
   b. first order
   c. second order
   d. none of these

2. Which one is the correct Monge's subsidiary equation by the definition $Rr + Ss + Tt = V$
   a. $Rdpdy + T\,dqdx + Vdxdy$
   b. $Rdpdy + T\,dqdx - Vdxdy$
   c. $Rdpdy - T\,dqdx + Vdxdy$
   d. All of above

3. The solution of PDE by elimination of arbitrary constants is
   $$z = A\,e^{pt}\sin px$$
   a. $\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta t^2} = 0$
   b. $\frac{\delta^2 z}{\delta x^2} - \frac{\delta^2 z}{\delta t^2} = 0$
   c. $\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} = 0$
   d. None of these

4. Which one of the following is correct and standard form of Lagrange's equation
   a. $Pp - Qq + R = 0$
   b. $Pp - Qq - R = 0$
   c. $Pp + Qq + R = 0$
   d. $Pp + Qq = R$

5. Which one of the following is correct Lagrange's characteristic equation form
   a. $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$
   b. $\frac{\delta x}{p} = \frac{\delta y}{q} = \frac{\delta z}{r}$
   c. $\frac{\delta x}{P} = \frac{\delta y}{Q} = \frac{\delta z}{R}$
   d. None of these

6. The Lagrange's Subsidiary equations of the PDE $zp = -x$ is
   a. $\frac{dx}{0} = \frac{dy}{0} = \frac{dz}{-x}$
   b. $\frac{dx}{z} = \frac{dy}{y} = \frac{dz}{-x}$
   c. $\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{x}$
   d. $\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$

7. A bounded solution to the partial differential equation
$\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2} + e^{-t}$ is

a. $u(x,t) = -e^{-t}$
b. $u(x,t) = x - e^{-t}$
c. a and b both are correct
d. None of these

8. If the partial differential equation is in the form of
$xyp + x^2yq = x^2y^2z^2$ then it is a

a. Linear
b. Semi Linear
c. Quasi Linear
d. Non linear

9. Partial Differential Equation $p \tan y + q \tan x = sec^2 z$ *is* of order

a. 1
b. 2
c. 0
d. None of these

10. If the partial differential equation is in the form of $p^2 + q^2 = 1$ then the equation is

a. Linear
b. Semi Linear
c. Quasi linear
d. Non linear

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Descriptive

Time : 1 hr. 15 mins. **Marks : 25**

[Answer question no.1 & any two (2) from the rest]

1. Find the equation of the integral surface of the differential equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$, which contains the line 5

$$x+y=0, z=1$$

2. a. Solve $\left\{\frac{b-c}{a}\right\} yzp + \left\{\frac{c-a}{b}\right\} zxq = \{\frac{a-b}{c}\}xy$ 5+5=10

   b. Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$orthogonally and which passes through the circle $x^2+y^2 = 1, z=1$

3. a. Solve and find the complete integral of $z = px + qy + pq$, 5+5=10

   b. Find the complete integral of

$$zpq = p + q$$

4. a. Derive the one dimensional wave equation by the method of separation of variables. 5+5=10

   b. Solve by Monge's method of one dimensional wave equation $r = a^2t$

5. Define the following terms 5

   a. Variational Problem

   b. Calculus of Variation

   c. Show that the functional 5

   $I_1[y(x)] = \int_a^b (y^1(x) + y(x))dx]$ is linear in the class $C^1[a.b]$, but the functional

   $I_2[y(x)] = \int_a^b [p(x)\{y^1(x)\}^2 + q(x)\{y(x)\}^2]dx$ is non-linear.

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