B.Sc. PHYSICS
SIXTH SEMESTER
MATHEMATICAL PHYSICS-III
BSP - 603A

[USE OMR FOR OBJECTIVE PART]

SET A

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. For the given periodic function $f(x) = x^3$ for $-\pi < x < \pi$ the coefficient a_n is
   a. 6.8968
   b. - 6.8968
   c. 0
   d. 0.7468

2. The Laplace transform of x^2
   a. $2/s^2$
   b. $2/s^3$
   c. $1/2s^2$
   d. $1/2s^3$

3. What are the conditions called which are required for a signal to fulfil to be represented as Fourier series?
   a. Dirichlet's conditions
   b. Gibbs phenomenon
   c. Fourier conditions
   d. Fourier transformation

4. The Lapace transform of $L\left[\frac{1}{x} f(x)\right]$ is
   a. $\int_0^s F(s)\,ds$
   b. $\int_s^\infty F(s)\,ds$
   c. $\int_0^\infty F(s)\,ds$
   d. $\int_{-\infty}^\infty F(s)\,ds$

5. Which of the following is an "even" function of t?
   a. t^2
   b. $t^2 - 4t$
   c. $\sin(2t) + 3t$
   d. $t^3 + 6$

6. The Laplace transform of $\sinh 2x$ will be
   a. $\frac{1}{s^2 - 2^2}$
   b. $\frac{s}{s^2 - 2^2}$
   c. $\frac{s}{s^2 + 2^2}$
   d. $\frac{2}{s^2 - 2^2}$

7. Fourier coefficient a_0 in Fourier series expansion of a function represents the
   a. Maximum value of the function
   b. 2 ×mean value of the function
   c. Minimum value of the function
   d. None of the mentioned

8. $L^{-1}\left(\frac{1}{2s+1}\right)$ will be

a. $\frac{1}{2}e^{-2x}$
b. $\frac{1}{2}e^{-x}$
c. $\frac{1}{2}e^{-\frac{x}{2}}$
d. $\frac{1}{2}e^{-\frac{x}{4}}$

9. In the following function f(x) is known as

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-isx} F(s) ds$$

a. Fourier transform of f(s)
b. Fourier transform of f(x)
c. Fourier transform of F(x)
d. Inverse Fourier transform of F(s)

10. $L^{-1}\left[\frac{1}{a}F\left(\frac{s}{a}\right)\right]$

a. $f(x)$
b. $f(x/a)$
c. $f(a\,x)$
d. $f(a/x)$

11. Fill in the blank. The property is known as---------, when F(s) is the complex Fourier transform of f(x) then $F\{f(x-a)\} = e^{isa} F(s)$

a. Shifting property
b. Change of scale property
c. Linear property
d. Modulation theorem

12. The Laplace transform of $x^2\ e^{-3x}$ is

a. $\frac{2!}{(s+3)^3}$
b. $\frac{2!}{(s+3)^2}$
c. $\frac{3!}{(s+3)^2}$
d. $\frac{3!}{(s+3)^3}$

13. Which of the following is the Fourier sine transform of f(x)?

a. $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \sin(st) dt$
b. $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F(s) \sin(sx) ds$
c. $F_s[f(x)] = \sqrt{\frac{\pi}{2}} \int_0^{\infty} F(s) \sin(sx) dx$
d. $F_s[f(x)] = \sqrt{\frac{\pi}{2}} \int_0^{\infty} f(t) \sin(st) dt$

14. Laplace transform of $y'(x)$ is

a. $L[f(x)] - sf(0)$
b. $s^2 L[f(x)] - f(0)$
c. $L[f(x)] - f(0)$
d. $s\,L[f(x)] - f(0)$

15. Fourier transform of f(t)=-----------x Laplace transform of g(t). Fill in the blank.

a. $\frac{1}{\sqrt{2\pi}}$
b. $\frac{1}{\sqrt{2\pi}}$
c. $\frac{1}{\sqrt{\pi}}$
d. None of these

16. The transform $L[x\, f(x)]$, where $L[f(x)] = F(s)$ will be

a. $\frac{d}{ds}(F(s))$
b. $-\frac{d}{ds}(F(s))$
c. $-\frac{d^2}{ds^2}(F(s))$
d. $\frac{d^2}{ds^2}(F(s))$

17. $F\{f''(x)\} = ?$

a. $(-is)^n F(s)$
b. $(is)^n F(s)$
c. $isF(s)$
d. $(is)^n F^n(s)$

18. If a function $f(z)$ is analytic and its derivative $f'(z)$ is continuous at all points inside and on a simple closed curve c, then $\oint f(z)\,dz$ will be

a. $2\pi i$
b. πi
c. 0
d. $\frac{\pi i}{2}$

19. If the Fourier series of $f(x)$ has only cosine terms then $f(x)$ must be

a. Odd function
b. Even function
c. Fundamental harmonic
d. Second harmonic

20. The value of $\oint \frac{z^2-z+1}{z-1}\,dz$, where $|z| = \frac{1}{2}$ will be

a. $2\pi i$
b. 0
c. $4\pi i$
d. $3\pi i$

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. If $L[f(t)] = F(s)$ and $g(t) = \begin{cases} f(t-a), & t > a \\ 0, & 0 < t < a \end{cases}$ then show that $L[g(t)] = e^{-as}F(s)$. Using this theorem find Laplace transform of 6+4=10

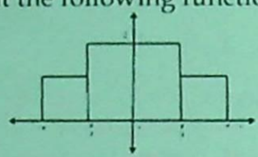
$$g(t) = \begin{cases} \cos\left(t - \frac{\pi}{3}\right), & t > \frac{\pi}{3} \\ 0, & 0 < t < \frac{\pi}{3} \end{cases}$$

2. Write the Dirichlet's condition for a Fourier series and represent the following function by a Fourier series:

$$f(t)=\begin{cases} t, & 0<t\le \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2}<t\le \pi \end{cases}$$

3. Find the Laplace transforms of (i) $f(t)=\frac{1}{2}t^2+t$, (ii) $f(t)=\frac{1}{a}\sinh(at)+\cos(at)$, (iii) $f(t)=\frac{1}{\sqrt{\pi t}}$. **4+6=10**

4. **a.** Represent the following function by a Fourier series: **4+6=10**



b. Expand the function $f(x)=x\sin x$, as a Fourier series in the interval $-\pi<x<\pi$.

5. State Cauchy Integral formula. Using this formula to evaluate $\oint \frac{z}{z^2-3z+2}\,dz$, where the closed curve is defined by $|z-2|=\frac{1}{2}$. **2+8=10**

6. Find Fourier transform of $f(x)=\begin{cases} 1-x^2, & |x|\le 1 \\ 0, & |x|>1 \end{cases}$. **10**

7. Using Laplace transforms, find the solution of the initial value problem: $y''(t)+4y'(t)=\cos t$, where $y(0)=2,\ y'(0)=0$. **10**

8. Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, x>0, t>0 subject to the condition **10**

   a. u=0 when x=0, t>0

   b. $u=\begin{cases} 1, 0<x<1 \\ 0, x\ge 1 \end{cases}$, when t=0

   c. u(x,t) is bounded.

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