REV-01
BSP/10/15

2024/05

SET A

B.Sc. PHYSICS
SIXTH SEMESTER
MATHEMATICAL PHYSICS-III
BSP - 603A

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. For the given periodic function $f(x) = x^3$ for $-\pi < x < \pi$ the coefficient a_0 is
   a. 6.8968 b. - 6.8968
   c. 0 d. 0.7468

2. The Laplace transform of x^2
   a. $2/s^2$ b. $2/s^3$
   c. $1/2s^2$ d. $1/2s^3$

3. What are the conditions called which are required for a signal to fulfil to be represented as Fourier series?
   a. Dirichlet's conditions b. Gibbs phenomenon
   c. Fourier conditions d. Fourier transformation

4. The Lapace transform of $\mathcal{L}\left[\frac{1}{t} f(t)\right]$ is
   a. $\int_0^s F(s)ds$ b. $\int_s^\infty F(s)ds$
   c. $\int_0^\infty F(s)ds$ d. $\int_{-\infty}^{\infty} F(s)ds$

5. Which of the following is an "even" function of t?
   a. t^2 b. $t^2 - 4t$
   c. $\sin(2t) + 3t$ d. $t^3 + 6$

6. The Laplace transform of $\sinh 2t$ will be
   a. $\frac{s}{s^2-2^2}$ b. $\frac{2}{s^2-2^2}$
   c. $\frac{3}{s^2-2^2}$ d. $\frac{2}{s^2-2^2}$

7. Fourier coefficient a_0 in Fourier series expansion of a function represents the
   a. Maximum value of the function b. 2 ×mean value of the function
   c. Minimum value of the function d. None of the mentioned

8. $L^{-1}\left(\frac{1}{2s+2}\right)$ will be

a. $\frac{1}{2}e^{-2x}$ b. $\frac{1}{2}e^{-x}$

c. $\frac{1}{2}e^{-\frac{1}{2}}$ d. $\frac{1}{2}e^{-\frac{1}{4}}$

9. In the following function f(x) is known as

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-isx}F(s)ds$$

a. Fourier transform of f(s) b. Fourier transform of f(x)

c. Fourier transform of F(x) d. Inverse Fourier transform of F(s)

10. $L^{-1}\left[\frac{1}{a}F\left(\frac{s}{a}\right)\right]$

a. $f(x)$ b. $f(x/a)$

c. $f(a\,x)$ d. $f(a/x)$

11. Fill in the blank. The property is known as---------, when F(s) is the complex Fourier transform of f(x) then $F\{f(x-a)\}=e^{isa}F(s)$

a. Shifting property b. Change of scale property

c. Linear property d. Modulation theorem

12. The Laplace transform of x^2e^{-3x} is

a. $\frac{2!}{(s+3)^3}$ b. $\frac{2!}{(s+3)^2}$

c. $\frac{3!}{(s+3)^2}$ d. $\frac{3!}{(s+3)^4}$

13. Which of the following is the Fourier sine transform of f(x)?

a. $F_s[f(x)]=\sqrt{\frac{2}{\pi}}\int_0^{\infty}f(t)\sin(st)dt$ b. $F_s[f(x)]=\sqrt{\frac{2}{\pi}}\int_0^{\infty}F(s)\sin(sx)ds$

c. $F_s[f(x)]=\sqrt{\frac{\pi}{2}}\int_0^{\infty}F(s)\sin(sx)dx$ d. $F_s[f(x)]=\sqrt{\frac{\pi}{2}}\int_0^{\infty}f(t)\sin(st)dt$

14. Laplace transform of $y'(x)$ is

a. $L[f(x)]-sf(0)$ b. $s^2L[f(x)]-f(0)$

c. $L[f(x)]-f(0)$ d. $sL[f(x)]-f(0)$

15. Fourier transform of f(t)=----------x Laplace transform of g(t). Fill in the blank.

a. $\frac{1}{\sqrt{2\pi}}$ b. $\frac{1}{\sqrt{2\pi}}$

c. $\frac{1}{\sqrt{\pi}}$ d. None of these

16. The transform $L[x\,f(x)]$, where $L[f(x)] = F(s)$ will be

a. $\frac{d}{ds}(F(s))$ b. $-\frac{d}{ds}(F(s))$

c. $-\frac{d^2}{ds^2}(F(s))$ d. $\frac{d^2}{ds^2}(F(s))$

17. $F\{f''(x)\} = ?$

a. $(-is)^n F(s)$ b. $(is)^n F(s)$

c. $isF(s)$ d. $(is)^n F''(s)$

18. If a function $f(z)$ is analytic and its derivative $f'(z)$ is continuous at all points inside and on a simple closed curve c, then $\oint f(z)dz$ will be

a. $2\pi i$ b. πi

c. 0 d. $\frac{\pi i}{2}$

19. If the Fourier series of $f(x)$ has only cosine terms then $f(x)$ must be

a. Odd function b. Even function

c. Fundamental harmonic d. Second harmonic

20. The value of $\oint \frac{z^2-z+1}{z-1}dz$, where $|z| = \frac{1}{2}$ will be

a. $2\pi i$ b. 0

c. $4\pi i$ d. $3\pi i$

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

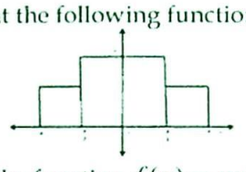
1. If $L[f(t)] = F(s)$ and $g(t) = \begin{cases} f(t-a), & t > a \\ 0, & 0 < t < a \end{cases}$ then show that $L[g(t)] = e^{-as}F(s)$. Using this theorem find Laplace transform of $g(t) = \begin{cases} \cos\left(t - \frac{\pi}{3}\right), & t > \frac{\pi}{3} \\ 0, & 0 < t < \frac{\pi}{3} \end{cases}$ **6+4=10**

2. Write the Dirichlet's condition for a Fourier series and represent the following function by a Fourier series:

$$f(t)=\begin{cases} t, & 0<t\le\dfrac{\pi}{2} \\ \dfrac{\pi}{2}, & \dfrac{\pi}{2}<t\le\pi \end{cases}$$

3. Find the Laplace transforms of (i) $f(t)=\frac{1}{2}t^2+t$, (ii) $f(t)=\frac{1}{a}\sinh(at)+\cos(at)$, (iii) $f(t)=\frac{1}{\sqrt{t}}$. 4+6=10

4. **a.** Represent the following function by a Fourier series: 4+6=10



b. Expand the function $f(x)=x\sin x$, as a Fourier series in the interval $-\pi<x<\pi$.

5. State Cauchy Integral formula. Using this formula to evaluate $\oint\frac{z}{z^2+3z+2}dz$, where the closed curve is defined by $|z-2|=\frac{1}{2}$. 2+8=10

6. Find Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases}$. 10

7. Using Laplace transforms, find the solution of the initial value problem: $y''(t)+4y'(t)=\cos t$, where $y(0)=2$, $y'(0)=0$. 10

8. Solve the equation $\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}$, x>0, t>0 subject to the condition 10

   a. u=0 when x=0, t>0

   b. $u=\begin{cases}1, 0<x<1\\ 0, x\ge 1\end{cases}$, when t=0

   c. u(x,t) is bounded.

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