REV-01
BSP/13/18

2023/12

**SET A**

# B.Sc. PHYSICS
## FIFTH SEMESTER
## QUANTUM MECHANICS & APPLICATIONS
## BSP – 501

[USE OMR FOR OBJECTIVE PART]

**Duration: 3 hrs.** **Full Marks: 70**

## (Objective)

**Time: 30 min.** **Marks: 20**

***Choose the correct answer from the following:*** ***1×20=20***

1. The probability density $|\Psi(x,t)|^2$ is defined as
   a. $\Psi(x,t)\Psi(x,t)$
   b. $\Psi^*(x,t)\Psi^*(x,t)$
   c. $\Psi^*(x,t)\Psi(x,t)$
   d. None of these

2. The operator representation of momentum in quantum mechanics is
   a. $-i\hbar\frac{\partial}{\partial x}$
   b. $i\hbar\frac{\partial}{\partial x}$
   c. $i\frac{\partial}{\partial x}$
   d. $-i\frac{\partial}{\partial x}$

3. A wave function has the form, $\psi(x,t) = Ae^{-ibt}$ ($A, b$ are real, independent of both $x$ and $t$). We conclude that
   a. The probability density is zero.
   b. The probability density oscillates with time.
   c. The probability density is a constant over time.
   d. The probability density decays with time.

4. The energy in the $n$th stationary state in an infinite square well potential varies with $n$ as
   a. $\frac{1}{n}$
   b. $n$
   c. $\frac{1}{n^2}$
   d. $n^2$

5. The energy difference between adjacent simple harmonic oscillator (1D) energy levels is
   a. $\hbar\omega$
   b. $2\hbar\omega$
   c. $3\hbar\omega$
   d. $\frac{1}{2}\hbar\omega$

6. The infinite square well potential has the form $V(x) = \begin{cases} 0, if\, 0 \le x \le a \\ \infty, otherwise \end{cases}$
   The probability of finding the particle within the range $-a \le x \le -2a$ is
   a. 1
   b. 0.5
   c. 0.25
   d. 0

7. $a_+$ is the raising operator and $\psi_0$ is the ground state for the simple harmonic oscillator. The correct choice is ($A$ is some constant)

a. $a_+\psi_0 = A\psi_1$

b. $a_+\psi_1 = A\psi_1$

c. $a_+\psi_0 = A\psi_0$

d. $a_+\psi_0 = 0$

8. The number of nodes in the $n$th state in an infinite square well potential is

a. $n$

b. $n-1$

c. $n+1$

d. $2n$

9. The solution to the Schrödinger equation for the free particle is given as

$$Ae^{i\left(kx-\frac{\hbar k^2}{2m}t\right)} + Be^{i\left(kx+\frac{\hbar k^2}{2m}t\right)}$$

Where $A, B$ are some constants and all other symbols have their usual meanings. The first and second terms represent

a. Left and right traveling waves, respectively

b. Right and left traveling waves, respectively

c. Left traveling waves

d. Right traveling waves

10. For a free particle in quantum mechanics, the group velocity $v_{group}$ and phase velocity $v_{phase}$ are related as

a. $v_{group} = 2v_{phase}$

b. $v_{group} = \frac{1}{2}v_{phase}$

c. $v_{group} = v_{phase}$

d. $v_{group} = 3v_{phase}$

11. The energy eigen value for a particle trapped in a cubical potential well of side 'a' for (1,1,1) state is

a. $\frac{9\pi^2\hbar^2}{2ma^2}$

b. $\frac{2\pi^2\hbar^2}{4ma^2}$

c. $\frac{3\pi^2\hbar^2}{2ma^2}$

d. $\frac{\pi^2\hbar^2}{9ma^2}$

12. Orbital angular momentum is classically expressed as

a. $\vec{L} = \frac{\vec{r} \times \vec{v}}{m}$

b. $\vec{L} = \frac{m\vec{r}}{\vec{v}}$

c. $\vec{L} = m(\vec{r} \times \vec{v})$

d. $\vec{L} = \frac{\vec{r}}{m\vec{v}}$

13. The magnitude of the spin magnetic dipole moment is given by

a. $\mu_z = \frac{1}{2}\frac{e\hbar}{m}$

b. $\mu_z = \frac{e\hbar}{4\pi m}$

c. $\mu_z = \frac{\sqrt{3}}{2}\frac{e\hbar}{m}$

d. $\mu_z = 0.866\hbar$

14. For L-shell (n=2), the number of possible sets of quantum numbers is

a. 2
b. 4
c. 6
d. 8

15. Quantization of orbital angular momentum is represented as

a. $L = n\pi\hbar$
b. $L = mvr/\hbar$
c. $L = n\hbar$
d. $L = n\pi/\hbar$

16. Which of the following is the expression for the energy of a particle in x-axis of a 3-dimensional potential box

a. $E = n_x\pi^2\hbar^2/8ma$
b. $E = n_x^2\pi^2\hbar^2/2a$
c. $E = n\pi\hbar/8ma$
d. $E = n_x^2\pi^2\hbar^2/2ma^2$

17. The separation between the Sodium D lines in terms of wavelength is

a. $\Delta\lambda = 0.6nm$
b. $\Delta\lambda = 6nm$
c. $\Delta\lambda = 60nm$
d. $\Delta\lambda = 600nm$

18. The spectral line series of H-atom which fall in visible range of wavelength is

a. Pfund
b. Bracket
c. Lyman
d. Balmer

19. The total energy E, contain in the radial wave equation of the H-atom, indicates the electron is bound to the proton, if

a. E is positive
b. E is negative
c. E is zero
d. E is imaginary

20. Energy in the first Bohr's orbit of H-atom is

a. 7.1 eV
b. 1.13 eV
c. - 4.01 eV
d. -13.6 eV

-- --- --

## Descriptive

**Time : 2 hrs. 30 mins.** **Marks : 50**

***[ Answer question no.1 & any four (4) from the rest ]***

**1.** The needle of a broken car speedometer is free to swing, and bounces perfectly off the pins at either end, so that if you give it a flick it is equally likely to come to rest at any angle between 0 to $\pi$. **1+2+2+3+2=10**

   **a.** What is the probability density, $\rho(\theta)$?

   **b.** Plot $\rho(\theta)$ as a function of $\theta$, from $\frac{-\pi}{2}$ to $\frac{3\pi}{2}$.

   **c.** Compute $\langle\theta\rangle$, $\langle\theta^2\rangle$ and $\langle\sigma\rangle$ for this distribution.

**2.** a. Draw the first three stationary states of the infinite square well (bounded between 0 to $a$). Identify the number of nodes in each of them. **4+3+3=10**

   b. A particle in the infinite square well has the initial wave function

$$\Psi(x,0) = A\sin^3(\pi x/a).$$

   (i) Normalize $\Psi(x,0)$. [No explicit integration is allowed]

   (ii) Find $\Psi(x,t)$.

**3.** The ladder operators are defined as $a_\pm \equiv \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$. **4+6=10**

   **a.** Find out the commutator between $a_-$ and $a_+$, that is

   **b.** Find the expectation value of the kinetic energy in the $n$th stationary state of the harmonic oscillator.

**4.** A free particle, which is initially localized in the range $-a < x < a$, is released at time $t = 0$: **2+3+2+3=10**

$$\Psi(x,0) = \begin{cases} A & \text{if} -a < x < a \\ 0 & \text{otherwise.} \end{cases}$$

Where $A$ and $a$ are positive real constants.

   **a.** Normalize $\Psi(x,0)$.

   **b.** Find $\phi(k)$ using the relation $\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Psi(x,0)e^{-ikx}dx$.

c. Construct $\Psi(x,t)$ following the relation

$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \phi(k) e^{-i\left(kx - \frac{\hbar k^2}{2m}t\right)} dk$. [Please note that you cannot solve it analytically. This could be done numerically. So just concentrate on the construction of the wave function.]

d. Discuss the limiting cases for $\Psi(x,0)$ and $\phi(k)$ ($a$ very large, and $a$ very small).

5. From time dependent Schrödinger's wave equation, using a variable separable method, deduce the time independent Schrödinger's wave equation — 10

6. a. What is Bohr's magnetron? Calculate the Bohr's magnetron for the ground state electron in H-atom (Given $\hbar = 1.05 \times 10^{-34}$ Js). — 1+4+2+3=10
   b. Deduce the magnitudes of the orbital dipole magnetic moment $\vec{\mu}_l$ of the electron in H-atom in p- and d-states. (Assume the spin of electron is zero)

7. Deduce the Schrödinger's equation for a Hydrogen atom in spherical polar coordinates and set the azimuthal wave equation applying separation of variables — 5+5=10

8. a. State and explain Pauli's exclusion principle. — 3+3+4 =10
   b. Discuss the distribution of quantum numbers in K- and L-shells, following Pauli's exclusion principle.

= = *** = =