2023/12

SET A

B.Sc. PHYSICS
FIRST SEMESTER
INTRODUCTION TO MATHEMATICAL PHYSICS
BSP - 101 IDMj [REPEAT]

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: 1×20=20

1. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$, then $div\ \vec{r}$ is
   a. 2 b. 3
   c. -3 d. -2

2. For the right-handed system of three coplanar vectors $\vec{A} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{B} = 3\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{C} = -\hat{i} + 4\hat{j} + m\hat{k}$, the value of m must be equal to
   a. 5 b. 8
   c. 0 d. 6.5

3. A vector points A vertically upward and point B towards north. The vector product A×B is
   a. along west b. along east
   c. zero d. vertically downward

4. The cross product D=E×F of vectors E= (1,-2, 3) and F=(4, 5, -6) is
   a. (-3, 18, 13) b. (-5, -18, -13)
   c. (-3, 18, -13) d. (5, -18, 13)

5. The line integral $\int_c$ F·dr represents
   a. The work done by the vector field F along the curve C
   b. The circulation of F along C
   c. The flux of F through C
   d. The divergence of F along C

6. If $\phi = yz$, then its gradient is
   a. $z\hat{j} + y\hat{k}$ b. 0
   c. $y\hat{j} + z\hat{k}$ d. $\hat{i} + \hat{j} + \hat{k}$

7. The electric field due to a point charge Q is expressed $\vec{E} = \frac{Q\hat{r}}{4\pi\varepsilon_0 r^2}$, then the divergence of electric field due to that point charge is

a. $\dfrac{3Q}{4\pi\varepsilon_0 r^2}$

b. $\dfrac{2Q}{4\pi\varepsilon_0 r}$

c. 0

d. $\dfrac{3Q}{4\pi\varepsilon_0 r}$

8. The direction of $grad\phi$ is

a. Tangential to level surfaces

b. Normal to level surface

c. Inclined at 45^0 to level surface

d. Arbitrary

9. If $\vec{A} = x\hat{i}$ and $\vec{B} = y\hat{j}$ then $\nabla(\vec{A}.\vec{B})$ is equal to

a. $x\hat{i} + y\hat{j}$

b. 0

c. $\frac{1}{2}yx^2\hat{i} + \frac{1}{2}xy^2\hat{j}$

d. 2

10. The flux leaving any closed surface per unit volume in a vector field $\vec{A}$ is called

a. $grad\,\vec{A}$

b. $div\,\vec{A}$

c. $curl\,\vec{A}$

d. $flux\,\vec{A}$

11. Which of the following vectors are perpendicular to each other?

(i) $2\hat{i} - 2j + 4\hat{k}$, (ii) $10\hat{i} + 8\hat{j} + 12\hat{k}$ and

(iii) $3\hat{i} + 11\hat{j} + 4\hat{k}$

a. (i) And (ii)

b. (ii) And (iii)

c. (iii) And (i)

d. None of these

12. If for two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then angle between $\vec{a}$ and $\vec{b}$ is

a. 0

b. $\dfrac{\pi}{2}$

c. $\dfrac{\pi}{4}$

d. $\dfrac{\pi}{3}$

13. If $\vec{F} = grad(2x^2 - 3y^2 + 4z^2)$, then $curl\,\vec{F}$ is

a. $4x - 6y + 8z$

b. $4x\hat{i} - 6yj + 8z\hat{k}$

c. 0

d. 3

14. Order of differential equation whose solution $y = ae^{x} + be^{2x} + ce^{3x}$ will be

a. 1
b. 2
c. 3
d. 0

15. $\frac{1}{f(D)}x^{m}$ will be equal to

a. $[F(D)]^{-1}x^{m}$
b. $F(D)x^{m}$
c. $mF(D)x^{m-1}$
d. $mx^{m-1}[F(D)]^{-1}$

16. What is the wronskian determinant of x^2, x^3

a. $2x^4$
b. x^4
c. $3x^4$
d. $4x^4$

17. The value of α so $e^{\alpha y^2}$ that is an I.F. of the equation $(e^{\frac{-y^2}{2}} - xy)dy - dx = 0$

a. -1
b. 1
c. $\frac{1}{2}$
d. $-\frac{1}{2}$

18. General solution of linear differential equation of first order $\frac{dx}{dy} + Px = Q$

a. $ye^{\int P.dx} = \int Qe^{\int P.dx}dx$
b. $xe^{\int P.dy} = \int Qe^{\int P.dy}dy + C$
c. $y = \int Qe^{\int P.dx}dx + C$
d. $x = \int Qe^{\int P.dy}dy + C$

19. Particular integral of $y'' + 2y' - 3y = e^{2x}$ is

a. $-\frac{1}{5}e^{2x}$
b. $\frac{1}{5}e^{2x}$
c. $-\frac{1}{5}$
d. $-\frac{1}{5}$

20. When y = f(x) + c g(x) is the solution of an ordinary differential equation then

a. f is called the particular integral (P.I.) and g is called the complementary function (C.F.)
b. f is called the complementary function (C.F.) and g is called the particular integral (P.I.).
c. f is called the complementary function (C.F.) and particular function (P.I.)
d. g is called the complementary function (C.F.) and particular function (P.I.)

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Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. 5+5=10

a. Show that the volume of the tetrahedron having $\vec{A}+\vec{B}$, $\vec{B}+\vec{C}$, $\vec{C}+\vec{A}$ as concurrent edges is twice the volume of the tetrahedron having $\vec{A}, \vec{B}, \vec{C}$ as concurrent edges.

b. If four points whose position vectors are **a, b, c, d** are coplanar, show that **[a b c] = [a d c] + [a d b] + [d b c]**

2. 4+4+2 =10

a. Prove that vector products can be expressed as a determinant.

b. Prove that **[a+b, b+c, c+a]** =2**[a,b,c]**.

c. Prove that work done is a scalar product.

3. 3+4+3 =10

a. Find the value of λ, for the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact.

b. Solve $\frac{d^2y}{dx^2} + 6y = sin4x$

c. $\frac{dx}{x} = tany\ dy$

4. 5+5=10

a. If $u = x + y + z, v = x^2 + y^2 + z^2, w = yz + zx + xy$, prove that $grad\ u, grad\ v, grad\ w$ are coplanar vector.

b. Find the value of n for which the vector $r^n \vec{r}$ is solenoidal, where $r = x\hat{i} + y\hat{j} + z\hat{k}$.

5. 4+3+3 =10

a. A fluid motion is given by $\vec{v} = (ysin\ z - \sin x)\hat{\imath} + (xsin\ z + 2yz)\hat{\jmath} + (xycos\ z + y^2)\hat{k}$. Is the motion irrotational?

b. If $y_1 = e^{-x}cosx, y_2 = e^{-x}sinx$ and $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y = 0$ then

(a) Verify that y_1 and y_2 satisfy the given differential equation.

(b) Apply Wronskian test to check that y_1 and y_2 are linearly independent.

6. a. Evaluate $curl\ (grad\ r^n)$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. 5+5=10

b. Find the constant a, b, c, so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.

7. a. If $\vec{A}$ is a constant vector and $\vec{R} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then prove that $curl[(\vec{A}.\vec{R})\vec{R}] = \vec{A} \times \vec{R}$ 5+5=10

b. If $\vec{F} = 2z\hat{\imath} - x\hat{\jmath} + y\hat{k}$, then evaluate $\iiint \vec{F}dv$, where v is the closed region bounded by the planes $x = 0, y = 0, x = 2, y = 4, z = x^2$ and $z = 2$.

8. a. Solve $(D^2 + 5D + 4)y = 2 - 3x$ 6+4=10

b. Prove that for every vector field $\vec{V}$, $div(curl\vec{V}) = 0$.

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