2023/12

SET A

B.Sc. PHYSICS
FIRST SEMESTER
INTRODUCTION TO MATHEMATICAL PHYSICS
BSP - 101 DMJ

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks:70**

(Objective)

Time: 30 min. **Marks:20**

Choose the correct answer from the following: ***1×20=20***

1. The position vector of a particle is denoted with $\vec{r}$. The velocity $\vec{r}$ would be
   a. $\frac{d\vec{r}}{dt}$ b. $\frac{d^2\vec{r}}{dt^2}$
   c. $\frac{d^3\vec{r}}{dt^3}$ d. $\frac{d^4\vec{r}}{dt^4}$

2. The condition for a vector $\vec{A}$ to be solenoidal is
   a. $\nabla\times\vec{A}=0$ b. $\nabla\cdot\vec{A}=0$
   c. $\nabla\vec{A}=0$ d. $\nabla^2\vec{A}=0$

3. The projection of a vector $\vec{A}$ along the x-direction is
   a. $\vec{A}\cdot\hat{x}$ b. $\vec{A}\cdot\hat{y}$
   c. $\vec{A}\cdot\hat{z}$ d. $\vec{A}\times\hat{x}$

4. Let $\vec{F}$ be the force on a particle moving along C. Then $\int_C \vec{F}\cdot d\vec{r}$ represents
   a. velocity of the particle
   b. projection of $\vec{F}$ in the direction of the position vector of the particle
   c. work done by the force
   d. acceleration of the particle

5. If $\nabla\times(\vec{A}\times\vec{B})=0$, then one of the followings is true.
   a. $\vec{A}$ is irrotational but not $\vec{B}$
   b. $\vec{B}$ is irrotational but not $\vec{A}$
   c. $\vec{A}\times\vec{B}$ is irrotational
   d. Both $\vec{A}$ and $\vec{B}$ are irrotational

6. $d(\vec{A}\times\vec{B})=$
   a. $d\vec{A}\times\vec{B}$ b. $d\vec{A}\times d\vec{B}$
   c. $\vec{A}\times d\vec{B}$ d. $d\vec{A}\times\vec{B}+\vec{A}\times d\vec{B}$

7. If m-1 and m+2 are factors of an auxiliary equation then general solution is
   a. $Ae^{-x}+Be^{2x}$ b. $e^{-x}+e^{2x}$
   c. $Ae^{x}+Be^{-2x}$ d. $e^{x}+e^{2x}$

8. In vector notation, a unit vector is denoted by
   a. $\hat{i}$ b. $\hat{j}$
   c. $\hat{k}$ d. All of them

9. The cross product of two parallel vectors is equal to

a. The product of their magnitudes
b. Zero
c. The sum of their magnitudes
d. The difference of their magnitudes

10. The elemental volume in the Cartesian coordinate is

a. $dxdydz$
b. dx
c. dy
d. dz

11. What is the wronskian determinant of x^2, x^3

a. $2x^4$
b. x^4
c. $3x^4$
d. $4x^4$

12. The complementary function of the differential equation $(D^2+6D+9)y=5e^{3x}$ is

a. $(C_1+C_2x)e^{-3x}$
b. $(C_1+C_2)e^{-3x}$
c. $(C_1+C_2x)e^{3x}$
d. $(C_1+C_2y)e^{3x}$

13. The gradient of a vector field is always

a. *Perpendicular to the vector field*
b. *parallel to the vector field*
c. *Tangential to the vector field*
d. *normal to the vector field*

14. Two differentiable function $Y_1(x)$ and $Y_2(x)$ are said to be linearly dependent if

a. $W(Y_1, Y_2\, x)=0$
b. $W(Y_1, Y_2\, x)\neq 0$
c. $W(Y_1, Y_2\, x)=1$
d. $W(Y_1, Y_2\, x)\neq 1$

15. $\frac{1}{f(D)}x^m$ will be equal to

A $[F(D)]^{-1}x^m$
B $F(D)x^m$
C $mF(D)x^{m-1}$
D $mx^{m-1}[F(D)]^{-1}$

16. If **A** and **B** are (3,4,5) and (6, 8,9) , then product of the vectors **A×B** is ----------

a. 3i+ 4j+ 9k
b. 3i-4j+9k
c. -4i+3j
d. 4i+3j

17. For the right handed system of three coplanar vectors $\vec{A}=\hat{i}-\hat{j}-2\hat{k}$, $\vec{B}=3\hat{i}+5\hat{j}+6\hat{k}$, $\vec{C}=-\hat{i}+4\hat{j}+m\hat{k}$, the value of m must be equal to

a. 5
b. 8
c. 0
d. 6.5

18. A vector points **A** vertically upward and point **B** towards north. The vector product **A×B** is

a. along west
b. along east
c. zero
d. vertically downward

19. General solution of linear differential equation of first order $\frac{dx}{dy} + Px = Q$

A. $ye^{\int P\,dx} = \int Qe^{\int P\,dx} dx$

b. $xe^{\int P\,dy} = \int Qe^{\int P\,dy} dy + C$

C. $y = \int Qe^{\int P\,dx} dx + C$

d. $x = \int Qe^{\int P\,dy} dy + C$

20. Which of the following is not the Axial vector

a. Torque

b. Angular Velocity

c. Angular Momentum

d. Acceleration

-- --- --

Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1 a. Prove that [a+b, b+c, c+a]=2[a,b,c]. 3+4+3 =10

b. Prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$

c. If $y_1 = e^{-x}\cos x,\ y_2 = e^{-x}\sin x$

Verify that the solutions satisfy the differential equation

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y = 0.$$

2. a. Determine the constant a so that the vector $\vec{V} = (x + 3y)\hat{\imath} + (y - 2z)\hat{\jmath} + (x + az)\hat{k}$ is solenoidal. 5+5=10

b. Solve the differential equation $\frac{d^2x}{dt^2} + \frac{g}{l}x = \frac{g}{l}L$ where g, l, L are constants subject to the conditions

$$x = a, \frac{dx}{dt} = 0 \text{ at } t = 0.$$

3. a. Evaluate $\nabla.\frac{\vec{r}}{r}$ 5+5=10

b. The acceleration of a particle at any time $t \geq 0$ is given by $\vec{a} = 12\cos t\ \hat{\imath} - 8\sin t\ \hat{\jmath} + 16t\hat{k}$.

If $\vec{v} = 0\ at\ t = 0\ and\ \vec{r} = 0,\ at\ t = 0,$ then find $\vec{v}$ and $\vec{r}$ at any time.

4. If $\vec{A} = (3x^2 + 6y)\ \hat{\imath} = 14yz\hat{\jmath} + 20xz^3\hat{k}$ evaluate $\int_C \vec{A}\cdot d\vec{r}$ along the following paths 4+6=10

i. $x = t, y = t^2, z = t^3.$

ii. the straight lines from (0,0,0) to (1,0,0), then to (1,1,0), and then to (1,1,1).

5. a. In curvilinear co-ordinate show that the differential of an arc length is 3+7=10

$$(ds)^2 = h_1^2(du)^2 + h_2^2(dv)^2 + h_3^2(dw)^2$$

b. Use divergence theorem to show that $\iint \nabla(x^2 + y^2 + z^2)\vec{ds} = 6V$ where S is the any closed surface enclosing volume V.

6. Solve: 5+5=10

i. $\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 10\frac{dy}{dx} = \sin x$

ii. $(D^3 + 1)y = e^{-x}$

7. a. An equation relating to stability of an areoplane is 5+5=10

$\frac{dv}{dt} = g\cos\alpha - kv$, where v is the velocity, g, α, k being constants.

Find an expression for the velocity if $v=0$ when $t=0$.

b. Find the constant a, b, c, so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.

8. a. Show that the volume of the tetrahedron having $\vec{A}+\vec{B},\ \vec{B}+\vec{C},\ \vec{C}+\vec{A}$ as concurrent edges is twice the volume of the tetrahedron having $\vec{A}, \vec{B}, \vec{C}$ as concurrent edges. 5+5=10

b. Solve $\frac{dy}{dx} + \frac{1}{x}y = x^3 - 3$

***