2023/12

M.Sc. MATHEMATICS
FIRST SEMESTER
ABSTRACT ALGEBRA I
MSM – 103

SET A

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(Objective)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1×10=10***

1. Let $U(n)$ be the set of all positive integers less than n and relatively prime to n. Then $U(n)$ is a group under multiplication modulo n. For $n = 248$, the number of elements in $U(n)$ is

   a. 180 b. 120
   c. 240 d. 60

2. Let G be a group. Suppose G has subgroups of order 45 and 75. If $|G| < 400$, then $|G|$ is equal to
   a. 90 b. 150
   c. 175 d. 225

3. The group $\mathbb{Z}_8^*$ is

   a. Cyclic and all of its subgroups are also cyclic.
   b. Non-cyclic but all of its subgroups are cyclic.
   c. Cyclic and some of its subgroups are cyclic.
   d. Non-cyclic but some of its subgroups are cyclic.

4. The number of elements in $\mathrm{Aut}(\mathbb{Z}_{25})$ is?
   a. 15 b. 20
   c. 25 d. 30

5. The value of for which $= \{, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56 is

   a. 5 b. 15
   c. 20 d. 25

6. Upto isomorphism, the number of Abelian group of order *121* is
   a. 1 b. 10
   c. 2 d. 11

7. Which of the following is/ are true?
   a. $\mathbb{Z}_3 \oplus \mathbb{Z}_7$ is isomorphic to $\mathbb{Z}_{21}$
   b. $\mathbb{Z}_4 \oplus \mathbb{Z}_6$ is isomorphic to $\mathbb{Z}_{24}$
   c. (10) is isomorphic to (12).
   d. (8) is isomorphic to (10).

8. Which of the following is/are false?

a. $_3$ is the smallest non-cyclic group.

b. All subgroups of $_3$ is cyclic.

c. $|(_3)| \neq 1$

d. None of these.

9. Let be a group of order 120 and let be a subgroup of . If $|| = 60$ then which of the following is always true

a. is a normal subgroup of .

b. is not a normal subgroup of .

c. may or may not be a normal subgroup of .

d. Data insufficient.

10. Consider the following statements:

P: Every finite cyclic group of order is isomorphic to $\mathbb{Z}$ and every infinite cyclic group is isomorphic to $\mathbb{Z}$.

Q: $\mathbb{Z}_{37}$ is a cyclic group.

R: Every Abelian group is cyclic but not conversely.

S: Every subgroup of a finite cyclic group is cyclic.

a. Only P is true

b. Only P and Q are true

c. Only P, Q and R are true

d. Only P, Q and S are true

-- --- --

Descriptive

Time : 1 hr. 15 min. Marks : 25

[Answer question no.1 & any four (4) from the rest]

1. Determine all homomorphisms from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{12}$. 5

2. a. Determine the number of elements of order 5 in $\mathbb{Z}_{25} \times \mathbb{Z}_{20}$. 5+3+2 =10
   b. Find all cyclic subgroup of S_3.
   c. Prove that - A_n is a normal subgroup of S_n.

3. a. Find all the generator of $U(25)$. 3+4+3 =10
   b. Show that - The center of a group G is a subgroup of G.
   c. Find the identity element of the group $\{5, 15, 25, 35\}$ under multiplication modulo 40.

4. a. Prove or disprove that- A_4 has no subgroup of order 6. 4+3+3 =10
   b. Find the number of Sylow 3-subgroups and Sylow 11-subgroups of a group of order 99.
   c. Prove or disprove that $U(12) \approx U(10)$.

5. a. Let σ and τ be the permutation defined by 4+3+3 =10

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 3 & 5 & 7 & 9 & 6 & 4 & 8 & 2 \end{pmatrix}$$
$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 3 & 4 & 9 & 6 & 5 & 2 & 1 \end{pmatrix}$$

   Find the inverse of σ & τ and find their order. Also, prove or disprove the followings:
   (i) σ and τ commute each other.
   (ii) $< \sigma > \cap < \tau >$ has order 1.

   b. Let $\beta \in S_7$ and suppose $\beta^4 = (2143567)$. Find β.

   c. Show that A_5 has 20 elements of order 3, and 15 elements of order 2.

= = *** = =