REV-01
MSM/01/05

2023/12

SET A

M.Sc. MATHEMATICS
THIRD SEMESTER
FUZZY SETS
MSM - 303E
[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(Objective)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1×10=10***

1. Let $U = (a, b, c, d)$ and $A = (b, d)$ be a crisp set on U. A can be expressed as a fuzzy set on U by
(numbers in the four sets representing membership grades of respective elements of U under A.)
   a. (0,0,1,1)
   b. (0,0,1,0)
   c. (0,1,0,1)
   d. (1,1,1,0)

2. Membership grade of a fuzzy set on a domain U is a number x such that
   a. $0 < x < 1$
   b. $0 \leq x < 1$
   c. $0 \leq x < \frac{1}{2}$
   d. $0 \leq x \leq 1$

3. For $0 < \alpha \leq 1$, let denote the -cut of a fuzzy set on a domain . For $_1 > _2$
   a. $_1 = _2$
   b. $_{\alpha_1} \subseteq A_{\alpha_2}$
   c. $A_{\alpha_1} \supseteq A_{\alpha_2}$
   d. None of these

4. Consider a fuzzy set A given by $A = (0, 0.9, 0.7, 0.4)$. Then level set $L(A)$ of A is
   a. $(0.7, 0.9, 0.4)$
   b. $(0.9, 0.4, 0.7)$
   c. $(0.9, 0.4, 0.7)$
   d. $(0.4, 0.7, 0.9)$

5. Let A be a fuzzy set of $\mathbb{R}$ defined by $A(x) = \begin{cases} 1 - e^{-x}, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0 \end{cases}$. Then the height of A is
   a. 0
   b. $\frac{1}{2}$
   c. 1
   d. 2

6. Let A be a fuzzy set of $\mathbb{R}$ defined by $A(x) = \begin{cases} 1 - e^{-x}, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0 \end{cases}$. Then support of A is
   a. $x \geq 0$
   b. $x > 0$
   c. $x \leq 0$
   d. ϕ.

7. Let A be a fuzzy set of $\mathbb{R}$ defined by $A(x) = \begin{cases} 1 - e^{-x}, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0 \end{cases}$. Then the core of A is
   a. $x \geq 0$
   b. $x > 0$
   c. $x \leq 0$
   d. None of these

8. Let A be a fuzzy set of $\mathbb{R}$ defined by $A(x) = \begin{cases} 1 - e^{-x}, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0 \end{cases}$. Then the fuzzy set A is

a. Normal
b. Subnormal
c. Both are true
d. Both are false

9. Let A_1 and A_2 be two fuzzy sets on domains U_1 and U_2 respectively. The cartesian product of fuzzy sets A_1 and A_2 is a fuzzy set $A = A_1 \times A_2$ defined by

a. $A(a_1, a_2) = \text{Min}\{A_1(a_1), A_2(a_2)\}$
b. $A(a_1, a_2) = \text{Max}\{A_1(a_1), A_2(a_2)\}$
c. $A(a_1, a_2) = \text{Min}\{1 - A_1(a_1), 1 - A_2(a_2)\}$
d. $A(a_1, a_2) = \text{Max}\{1 - A_1(a_1), 1 - A_2(a_2)\}$

10. Let $U = \{2,4,6\}, V = \{1,3\}$ be two sets of natural numbers. R will be a fuzzy relation from U to V if $R(u, v)$, the membership grade of (u, v) under R is given by

a.

R	1	3
2	0	1/2
4	2/3	1
6	−1	2

b.

R	1	3
2	1	0
4	1/2	−1/2
6	2	1

c.

R	1	3
2	1/3	1/2
4	0	1
6	1/5	1/4

d.

R	1	3
2	−1	0
4	2	1
6	0	1/2

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(Descriptive)

Time : 1 hr. 15 mins. Marks : 25

[Answer question no.1 & any two (2) from the rest]

1. Explain briefly the motivation behind the definition of fuzzy sets. Explain the concepts of interval valued Fuzzy sets and Fuzzy sets of Type 2 with the help of diagrams. 1+2+2 =5

2. a. When is a fuzzy set A on $\mathbb{R}$ said to be convex? Prove that a fuzzy set A on $\mathbb{R}$ is convex if and only if 1+4+2+3 =10
$A(\lambda x_1 + (1-\lambda)x_2) \geq \text{Min}\{A(x_1), A(x_2)\}$.

   b. Define α-cut and strong α-cut for a fuzzy set A defined on a universal set U. Let A be a fuzzy set on $U = \{a, b, c, d\}$ defined by
$$A = \frac{0.8}{a} + \frac{1.0}{b} + \frac{0.3}{c} + \frac{0.1}{d}$$
Denoting the α-cut and strong α-cut for A by A_α and $A_{+\alpha}$ respectively find $A_{0.2}$, $A_{0.3}$, $A_{+0.3}$, $A_{0.9}$, $A_{+0.9}$, $A_{+1.0}$

3. a. Define the following terms for a fuzzy set A with illustrations by examples scalar Cardinality of A, Height of A, Core of A and Support of A. 4+3+3 =10

   b. What do you know by normalization of a fuzzy set A? Normalise the fuzzy sets defined below –
(i) $A = \frac{0.2}{a} + \frac{0.5}{b} + \frac{6}{c} + \frac{0.8}{d}$
(ii) $B = \frac{0.6}{a} + \frac{0.4}{b} + \frac{0.5}{c} + \frac{0.3}{d}$

4. a. Explain the concept of fuzzification of a fuzzy set A. 5+5=10

Let $U = \{a, b, c\}$ and $A = \frac{0.3}{a} + \frac{0.6}{b} + \frac{0.7}{c}$. Also let K_a, K_b and K_c be defined on U by $K_a = \frac{0.7}{a} + \frac{0.4}{b}$, $K_b = \frac{0.4}{a} + \frac{1.0}{b} + \frac{0.4}{c}$ and $K_c = \frac{0.2}{b} + \frac{0.8}{c}$

Compute $F(A)$, the fuzzification of the set fuzzy set A.

b. Let $A_1, A_2, \cdots, A_n$ be n fuzzy sets on n domains $U_1, U_2, \cdots, U_n$ respectively. Define a fuzzy set $A = A_1 \times A_2 \times \cdots \times A_n$ on the domain $U_1 \times U_2 \times \cdots \times U_n$.

Demonstrate $A = A_1 \times A_2$ on $U_1 \times U_2$

where $U_1 = \{a, b, c\}$, $U_2 = \{x, y\}$

$A_1 = \frac{0.2}{a} + \frac{0.7}{b} + \frac{0.5}{c}$, $A_2 = \frac{0.5}{x} + \frac{0.3}{y}$

5. a. What is an n-ary fuzzy relation on $U_1 \times U_2 \times \cdots \times U_n$, where $U_1, U_2, \cdots, U_n$ are any n domains? Give an example of a binary fuzzy relation. 2+3+5 =10

b. Let R be a fuzzy relation on $U \times V$ and α be such that $0 < \alpha \leq 1$. Denote the α-cut of R by $R_\alpha = \{(u, v) \mid R(u, v) \geq \alpha\}$.
Verify the decomposition theorem for R i.e., $R = \sum \alpha(R_\alpha)$, by taking $R = \begin{bmatrix} 0.7 & 0.4 \\ 0.4 & 0.0 \end{bmatrix}$. Where R_α is an α-cut of R for $0 < \alpha \leq 1$.

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