SET A

M.Sc. MATHEMATICS
THIRD SEMESTER
ORDINARY DIFFERENTIAL EQUATION-I
MSM – 303

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(Objective)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1×10=10***

1. An example of non-homogeneous Equation is
   a. $y'' + y' - 7y = 0$
   b. $y'' + y' - 7y = 8x$
   c. $y'' - y' - 7y = 0$
   d. $y'' + y' + 9y = 0$

2. In Picard's Method n^{th} approximation is given by
   a. $y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\,dx$
   b. $y_n = y_0 + \int_{x_0}^{x} f(x, y_n)\,dx$
   c. $y_{n-1} = y_n + \int_{x_0}^{x} f(x, y_{n-1})\,dx$
   d. $y_n = y_{n-1} + \int_{x_0}^{x} f(x, y_{n-1})\,dx$

3. In homogeneous method the dependent variable x can be replace by
   a. $z = e^{x}$
   b. $x = e^{-z}$
   c. $z = e^{-x}$
   d. $x = e^{z}$

4. Solution of a system of Ordinary Differential Equation $\frac{dx}{dt} = \alpha x$ is
   a. $x(t) = ce^{\alpha t}, if \alpha > 0 |x(t)| \to 0 as t \to \infty$
   b. $x(t) = ce^{\alpha t}, if \alpha < 0 |x(t)| \to 0 as t \to \infty$
   c. $x(t) = ce^{\alpha t}, if \alpha < 0 |x(t)| \to 0 as t \to \infty$
   d. $x(t) = ce^{\alpha t}, if \alpha > 0 |x(t)| \to 0 as t \to \infty$

5. In Picard's Method, for the equation $\frac{dy}{dx} = 2 - \frac{y}{x}$, where $y = 2$ when $x = 1$ the function $f(x, y_{n-1}) = ?$
   a. $2 - \frac{y_n}{x}$
   b. $1 - \frac{y_{n-1}}{x}$
   c. $2 - \frac{y_{n-1}}{x}$
   d. $\frac{y_{n-1}}{x}$

6. Given a non-homogeneous equation $\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=Q(x).If Q(x)$ is $\sin 2x$,then trial solution is

a. $y=ax^2+bx+c$

b. $y=c_1\cos 2x+c_2\sin 2x$

c. $y=ae^x$

d. $y=c_1\sin 2x+c_2\cos 2x$

7. Clairaut's Equation is

a. $y=xp+f(p), p=\frac{\partial y}{\partial x}$

b. $y=xp+f(p), p=\frac{dy}{dx}$

c. $y=xp+f(p,q), p=\frac{dy}{dx}$

d. $y=xp+f(p,q), p=\frac{\partial y}{\partial x}$

8. Suppose a non-homogeneous differential equation is $\left(\frac{d^2}{dx^2}+2x^2\right)g(x)=f(x)$,green's function is

a. $\frac{dy}{dx}+\lambda y=0$

b. $\left(\frac{d^2}{dx^2}+2x^2\right)^{-1}$

c. $\left(\frac{d^2}{dx^2}+2x^2\right)$

d. $\left(\frac{d^2}{dx^2}+2x^2\right)g(x)$

9. One of the application of Green's function is

a. To Solve non-homogeneous boundary value problems

b. To solve homogeneous boundary value problems

c. To solve Linear BVP

d. None of the above

10. Eigen values for the matrix A= $\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$ is

a. $\lambda=-1,-6$

b. $\lambda=1,-6$

c. $\lambda=-1,6$

d. $\lambda=1,6$

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Descriptive

Time : 1 hr. 15 mins. Marks : 25

[Answer question no.1 & any two (2) from the rest]

1. State Uniqueness Theorem Prove that 1+4=5

$$y_n = y_0 + \sum_{n=1}^{n} (y_n -- y_{n-1})$$ must be continuous

2. Find the third approximation of the solution of the equation 10

$$\frac{d^2y}{dx^2} = x^3\left(y + \frac{dy}{dx}\right) \text{ where } y = 1, \frac{dy}{dx} = 1/2 \text{ when } x = 0$$

3. What do you mean by Lipschitz condition and constant? Show that the function $f(x,y) = y^{1/2}$ does not satisfy the Lipschitz condition on the rectangle $R: |x| \le 1, |y \le 1|$. What is Singular solution? Find the singular solution of $p^3 - 4xyp + 8y^2 = 0$ 2+3+2+3=10

4. Solve the Differential equation and find the eigen function and corresponding eigen values of $X'' + \lambda X = 0$ with the boundary condition $X(0) = 0, X'(L) = 0$ 4+6=10

5. Solve 5+5=10

(a) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$

(b) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$

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