REV-01
MSM/34/39

2023/06

SET A

M.Sc. MATHEMATICS
FOURTH SEMESTER
DYNAMICAL SYSTEM
MSM - 404 B
[USE OMR FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

(Objective)

Time : 30 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. Sink is an equilibrium point up to which the solution curve of a system
   a. Seems to converge as $t \rightarrow \infty$
   b. Sufficiently close as $t \rightarrow \infty$
   c. Asymptotic as $t \rightarrow \infty$
   d. None

2. Sink is also known as
   a. Attractors
   b. Repellers
   c. Asymptotically stable solutions.
   d. None

3. If $P(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots\ldots\ldots\ldots a_{n-1}\lambda + a_n$ then all the roots of the polynomial $P(\lambda)$ are negative or have negative real part iff the determinants of the all Hurwitz matrices are
   a. Positive
   b. Negative
   c. Zero
   d. None

4. If x^* is an equilibrium point for the equation $\frac{dx}{dt} = f(x)$ then, x^* is a sink if
   a. $f'(x^*) > 0$
   b. $f'(x^*) = 0$
   c. $f'(x^*) < 0$
   d. None

5. An equilibrium is asymptotically stable
   a. If all eigen values have negative real parts
   b. If at least one eigen value has positive real part
   c. If all the eigen values of the Jacobian matrix have non-zero real parts.
   d. If at least one eigen value of the Jacobian matrix is zero or has a zero real part

6. The equilibrium is said to be hyperbolic
   a. If all eigen values have negative real parts
   b. If at least one eigen value has positive real part
   c. If all the eigen values of the Jacobian matrix have non-zero real parts.
   d. If at least one eigen value of the Jacobian matrix is zero or has a zero real part

7. If the coefficients of the characteristic polynomial $P(\lambda)$ are positive then roots of the characteristic polynomial are

a. negative
b. Having negative real part
c. Both A and B
d. None

8. The linear system of equation

$$\dot{x} = 2x + y$$
$$\dot{y} = 3x + 4y$$

have

a. Stable source
b. Unstable node
c. Saddle point
d. None

9. For a linearised system of dynamical system if the eigen values λ_1 and λ_2 are purely imaginary then origin is

a. Center
b. Spiral
c. May be both A and B
d. None

10. The linear system of equation

$$\dot{x} = y$$
$$\dot{y} = -2x - 2y$$

have

a. Spiral source
b. Spiral sink
c. Saddle point
d. None

11. If

$$x' = -x - x^2$$

$$y' = -y - x$$

Then basin of attraction is

a. The whole real plan
b. $y > -1$
c. $x > \frac{1}{2}$
d. None

12. The first order partial differential equations $p = P(x, y)$, $q = Q(x, y)$ are compatible if

a. $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial Y}$
b. $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial Y}$
c. Both a. and b.
d. None

13. $\frac{\partial z}{\partial x} = 5x - 7y$

$\frac{\partial z}{\partial y} = 6x + 8y$ possess

a. Common solution
b. No common solution
c. No solution
d. None

14. For a particular system of equations if V is a positive definite Lyapunov function , then by Lyapunov Theorem the system is globally stable only when

a. $\dot{V} > 0$
b. $\dot{V} < 0$
c. Both a. and b.
d. None

15. For a system if both the eigen values of the Jacobian matrix are of opposite sign then it will give

a. Saddle point
b. Spiral
c. Lines of equillibrium
d. None

16. For a system if both the eigen values of the Jacobian matrix are zero then system is called

a. Stable
b. Unstable
c. Algebraically stable
d. None

17. For a system if one eigen value of the Jacobian matrix is zero and other is non-zero then system has

a. Spiral source
b. Spiral Sing
c. Whole lines of equillibrium
d. None

18. If the Jacobian matrix of a system has complex eigen vales having negative real part then the system shows

a. Stable spiral
b. Unstable spiral
c. Stable source
d. Stable sink

19. If the Jacobian matrix of a system has complex eigen vales having positive real part then the system shows

a. Stable spiral
b. Unstable spiral
c. Stable source
d. Stable sink

20. If the Jacobian matrix of a system has complex eigen values having zero real part then the system shows

a. Stable center
b. Neutral Center
c. Both a. and b.
d. None

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Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Apply Lyapunov method to establish global stability 10

$$\dot{G} = -aG - bI + \alpha E + \delta$$
$$\dot{I} = cGI - dI$$
$$\dot{E} = \beta E(1 - \gamma E)$$

2. a. Write the Poincare Eigen value criterion for stability of a system 5+5=10
   b. Show that the solutions $\phi_1(x) = e^{2x}$, $\phi_2(x) = xe^{2x}$ and $\phi_3(x) = x^2e^{2x}$ are linearly independent solutions of $y''' - 6y'' - 8y = 0$ on the interval 0≤x≤1

3. a. State and prove Bendixon-Dulac theorem for limit cycle. 5+5=10
   b. State and prove Bendixon-Bendixon theorem for limit cycle.

4. a. Find basin of attraction for the following system 5+5=10

$$x' = -x - x^3$$
$$y' = -y$$

   b. Check whether the following system has closed trajectories or not

$$x' = y + y^2e^x$$
$$y' = x$$

5. a. State the complete Routh Hurwitz Theorem, mentioning its significance. 5+5=10
   b. State Lyapunov theorem, mentioning its significance.

6. a. State and prove Bendixon theorem for limit cycle 5+5=10

b. Show that linearly independent solution of $y''' - 2y' + 2y = 0$ are $e^x Sinx$ and $e^x Cosx$. What is the general solution? Find the solution of $y(x)$ with the property $y(0) = 2$, $y'(0) = -3$

7. a. Verify the existence of limit cycles of the following system 5+5=10

$$x' = ax + by$$
$$y' = cx + dy$$

b. Is the origin for the following system stable or asymptotically stable, Find the basin of attraction

$$x' = y - x^3$$
$$y' = -x - y^3$$

8. Establish the stability of the following system of equations using 5+5=10

i. Eigen value method
ii. Lyapunov method

$$\dot{G} = -aG - bI + \alpha E + \delta$$
$$\dot{I} = cG - dI$$
$$\dot{E} = \beta E(1 - \gamma E)$$

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