REV-01
MSM/49/25/30

2023/06

SET B

M.Sc. MATHEMATICS
FOURTH SEMESTER
GRAPH THEORY
MSM – 401
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. Full Marks: 70

(PART-A: Objective)

Time: 30 min. Marks: 20

Choose the correct answer from the following: ***1X20=20***

1. A connected undirected graph containing n vertices and n −1 edges ....
   a. Cannot have cycles
   b. Must contain at least one cycle
   c. Can contain at most two cycles
   d. Must contain at least two cycles

2. What is the total number of edges of a k− regular graph with n vertices
   a. kn
   b. $\frac{kn}{2}$
   c. k+ n
   d. kn^2

3. I. Every bipartite graph contains an odd cycle.
   II. Every tree is a bipartite graph.
   a. I is true and II is false
   b. I is false and II is true
   c. I and II are false
   d. I and II are true

4. If a graph is planar, then it is embeddable on a
   a. Circle
   b. Square
   c. Sphere
   d. Plane

5. In a 2-connected graph G, any two longest cycles have at least – – vertices in common.
   a. 0
   b. 1
   c. 2
   d. 3

6. P: Let G_1, G_2 and G_3 be simple graphs. If $G_1 \cong G_2$ and $G_2 \cong G_3$ then $G_1 \cong G_3$.
   Q: If G_1 and G_2 are isomorphic then they have a common edge.
   a. P is true and Q is false
   b. P is false and Q is true
   c. P and Q are false
   d. P and Q are true

7. Total number of edges of a complete graph $K_{m,n}$
   a. $m + n$
   b. $m - n$
   c. mn
   d. None of these

8. A simple graph has – –
   a. Loops
   b. parallel edges
   c. loops and parallel edges
   d. None of these

9. If a graph G of n vertices is simple and $\delta \geq n-1$, then G is
   a. Complete
   b. Bipartite
   c. Connected
   d. Planar

10. What is the maximum number of edges in a bipartite graph having 12 vertices.
a. 24
b. 38
c. 36
d. 32

11. What is the number of edges present in complete graph K_n having n vertices?
a. $\frac{n(n+1)}{2}$
b. $\frac{n(n-1)}{2}$
c. n^2
d. None of these

12. Two graphs G & H are isomorphic if
a. G & H have same number of vertices
b. G & H have same same number of edges
c. Whenever two vertices are adjacent in G their respective images are also adjacent in H.
d. All of a,b & c are true

13. A Graph containing m edges can be decomposed in
a. $2^m - 1$ ways
b. $2^{m-1} - 1$ ways
c. 2^m ways
d. None

14. The number of Hamiltonian Circuits in a complete graph of n vertices
a. $\frac{(n-1)!}{2}$
b. $\frac{n!}{2}$
c. $\frac{(n-2)!}{2}$
d. None of these.

15. What is the determinant of the adjacency matrix of $C4$
a. 1
b. -1
c. 0
d. None of this

16. Let $\kappa(G)$ = vertex connectivity, $\lambda(G)$ = edge connectivity and $\delta(G)$ = minimum degree of graph G. Then
a. $\kappa(G) \le \delta(G) \le \lambda(G)$
b. $\lambda(G) \le \delta(G) \le \kappa(G)$
c. $\delta(G) \le \kappa(G) \le \lambda(G)$
d. $\kappa(G) \le \delta(G) < \lambda(G)$

17. Let G be a simple graph where every pair of vertices is connected. Then G is
a. Trivial
b. Complete
c. Disconnected
d. Self-complementary

18. If each and every vertex in G has degree at most 23 then G can have a vertex colouring of ________
a. 24
b. 23
c. 176
d. 54

19. Which of the following statement is/are TRUE?
P: A cycle is walk whose end vertices are same.
Q: A cycle is a path whose end vertices are same.
a. P only
b. Q only
c. Both P and Q
d. Neither P and Q

20. A graph is called a ____if it is a connected acyclic graph
   a. Cyclic graph
   b. Regular graph
   c. Tree
   d. Trivial graph

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

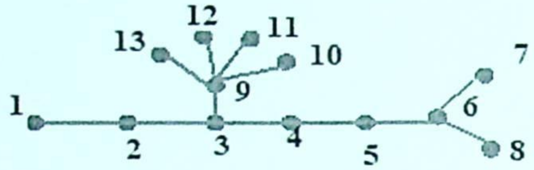
1. a. Prove that the maximum number of edges among all p point graph with no triangle is $\left[\frac{p^2}{4}\right]$, ($[x]$denotes the greatest integer not exceeding the real number x) **5+5=10**
   b. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$ and also prove that any self-complementary graph has $4n$ or $4n+1$ vertices.

2. a. Calculate the number of vertices and number of edges of the graph $G_1\ (p_1, q_1)$ and $G_2\ (p_2, q_2)$ graph by the rule of $G_1 + G_2$, $G_1 \times G_2$ **5+5=10**
   b. Define Pseudo Graph and Quasi- transitive dia-graph. Show that every circuit has an even number of edges in common with any cut-set.

3. Write down the five properties of TREE. Define binary tree and spanning tree explain with their properties and a diagram. **10**

4. a. Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-set. **5+5=10**
   b. Prove that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph

5. a. Find the eccentricity of each vertices and Centre of the following graph 5+5=10



b. Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges

6. a. If G is a plane (p, q) graph with r faces and k is components then show that $p - q + r = k + 1$ 5+5=10

b. If G is a plane (p, q) plane graph in which every face s an n cycle then show that $q = \frac{n(p-2)}{n-2}$

7. a. If G is a Tree with a point, $n \geq 2$ then show that the chromatic polynomial of tree is $f(G, \lambda) = \lambda\,(\lambda - 1)^{n-1}$ 5+5=10

b. Find the Chromatic polynomial of $K_5 - 2x$

8. Show that the following statements are equivalent for any graph G 10

a. G is two colourable

b. G is bi-partite

c. Every cycles of G has even length.

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