REV-01
BSP/07/12

2023/06

SET A

B.Sc. PHYSICS
SIXTH SEMESTER
MATHEMATICAL PHYSICS
BSP – 603A
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. The magnitude of the complex number $\frac{1}{\sqrt{2}}(-1+i)$ is
   a. -1 b. 1
   c. 0 d. 2

2. The value of i^9 is
   a. 1 b. i
   c. $-i$ d. -1

3. The magnitude of the difference of the complex numbers $(4+2i)$ and $(5+3i)$ is
   a. $\sqrt{2}$ b. 1
   c. 2 d. 0

4. The reciprocal of the complex number $(4-3i)$ is
   a. $\frac{1}{25}(-4-3)$ b. $\frac{1}{25}(-4+3i)$
   c. $\frac{1}{25}(4-3i)$ d. $\frac{1}{25}(4+3i)$

5. The Laplace transform $f(t)=t^0$ is
   a. s^{-1} b. s^{-2}
   c. s^1 d. s^2

6. If L[f(x)]=F(s) then L[cos 2 x] is
   a. $\frac{2s}{s^2+4}$ b. $\frac{2s}{s^2-4}$
   c. $\frac{2s}{s^2+2}$ d. $\frac{s}{s^2+2}$

7. If L[F(x)]=f(s) then L[$x^n F(x)$] is
   a. $(-1)^n f^{n+1}(s)$ b. $(-1)^n f^{n-1}(s)$
   c. $(-1)^n f^n(s)$ d. $(-1)^n f^{-n+1}(s)$

8. $\frac{1}{3!}L^{-1}(1/s^4)$ is
   a. x^1 b. x^2
   c. x^3 d. x^4

9. The value of $\oint \frac{e^z}{z+1}\,dz$ for a circle of |z|=1 is
   a. $2\pi i$ b. πi
   c. 0 d. 1

10. The value of $\oint \frac{dz}{z-a}$ for a circle of |z-a|=r is

a. 0 b. $2\pi i$

c. πi d. 2π

11. What are the conditions called which are required for a signal to fulfill to be represented as Fourier series?

a. Dirichlet's conditions b. Gibbs phenomenon

c. Fourier conditions d. Fourier phenomenon

12. Choose the condition from below that is not a part of Dirichlet's conditions?

a. It is single-valued, if the function f(x) for the interval (-π,π) b. It is bounded, if the function f(x) for the interval (-π,π)

c. It has only a finite number of discontinuous, if the function f(x) for the interval (-π,π) d. It is a periodic signal, if the function f(x) for the interval (-π,π)

13. For the given periodic function $f(x) = x^3$ for $-\pi < x < \pi$ the coefficient a_0 is

a. 6.8968 b. -6.8968

c. 0 d. 0.7468

14. A function f(x) is called skew symmetric function if

a. $f(-x) = -f(x)$ b. $f(-x) = f(x)$

c. $f(-x) = -f(-x)$ d. $f(-x) = 0$

15. If the Fourier series of $f(x)$ has only cosine terms then $f(x)$ must be

a. Odd function b. Even function

c. Fundamental harmonic d. Second harmonic

16. Which of the following is the Fourier sine transform of f(x)?

a. $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^x f(t)\sin(st)dt$ b. $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^x F(s)\sin(sx)ds$

c. $F_s[f(x)] = \sqrt{\frac{\pi}{2}} \int_0^x F(s)\sin(sx)dx$ d. $F_s[f(x)] = \sqrt{\frac{\pi}{2}} \int_0^x f(t)\sin(st)dt$

17. At what condition $F\{f'(x)\} = isF(s)$

a. $f(x) \to 0$ as $x \to \pm\infty$ b. $f(x) \to \infty$ as $x \to 0$

c. $f(x) \to \infty$ as $x \to \pm\infty$ d. none of these

18. Fill in the blank. The property is known as---------, when F(s) is the complex Fourier transform of f(x) then $F\{f(x-a)\} = e^{isa}F(s)$

a. Shifting property
b. Change of scale property
c. Linear property
d. Modulation theorem

19. $F\{f^n(x)\} = ?$

a. $(-is)^n F(s)$
b. $(is)^n F(s)$
c. $isF(s)$
d. $(is)^n F^n(s)$

20. Fourier transform of f(t)=----------x Laplace transform of g(t). Fill in the blank

a. $\frac{1}{\sqrt{2\pi}}$
b. $\frac{1}{\sqrt{2\pi}}$
c. $\frac{1}{\sqrt{\pi}}$
d. None of these

-- --- --

(Descriptive)

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. Solution the differential equation $\frac{d^2y}{dt^2} + y(t) = t$ using Laplace transform method where the boundary conditions are y (t=0) = = and $y'(t = 0) = 1$. **7+3=10**

b. The Laplace transform of $a\,x^2 + b\,x^3$ will be?

2. a. Write the Dirichlet's condition for a Fourier series. **2+3+3+2=10**

b. Determine the Fourier coefficient a_0 and a_n .

c. What do you mean by fundamental harmonic of Fourier series.

3. a. If n is a positive integer, prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$ **4+3+3=10**

b. If ω is a cube root of unity then find $(1+\omega)^6$.

c. Express $\frac{1}{2}e^{\frac{i\pi}{3}}$ in the complex number $a + i\,b$.

4. What do you mean by even function of a Fourier series? Expand the function $f(x) = x\sin x$, as a Fourier series in the interval $-\pi < x < \pi$. Hence deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \ldots = \frac{\pi - 2}{4}$. **1+5+4=10**

5. a. Using Cauchy's integral formula evaluate $\oint \frac{z}{z^2-3z+2}\,dz$ for a circle |z-2|=1/2. **4+3+3=10**

b. Find the inverse transform of $\frac{2}{(s-2)^2+4}$.

c. Find the Laplace transform L [F(t)] if

$$F(t)=\begin{cases}\cos(t-\frac{\pi}{4}), & t>\frac{\pi}{4}\\ 0, & t<\frac{\pi}{4}\end{cases}$$

6. 6+4=10

a. If $F_c(s) = \frac{1}{2}\tan^{-1}(\frac{2}{s^2})$, find $f(x)$.

b. Establish the relationship between Fourier and Laplace transforms.

7. 4+2+4 =10

a. Find the value of a & b if $\frac{(1+i)\,a-2\,i}{3+i} + \frac{(2-3\,i)\,b+i}{3-i} = i$.

b. Find the value of $\sqrt{i} + \sqrt{-i}$.

c. Find the complex number if $\arg(z+1)=\frac{\pi}{6}$ and $\arg(z-1)=\frac{2\pi}{3}$.

8. Find Fourier sine and cosine transform of x^{n-1}. 5+5=10

= = *** = =