REV-01
BSP/07/12

2023/06

SET A

B.Sc. PHYSICS
SIXTH SEMESTER
ELECTROMAGNETIC THEORY
BSP – 601
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(**Objective**)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. $f(z,t)$ represents a wave of fixed shape traveling in the z direction at speed v. One of the following forms does not represent a wave. Which one?
   a. $Ae^{-b(z-vt)^2}$ b. $Ae^{b(bz^2+vt)}$
   c. $\frac{A}{b(z-vt)^2}$ d. $A\sin[b(z-vt)]$

2. For an electromagnetic plane wave $\hat{n}$ and $\hat{k}$ are the polarization vector and the direction of wave vector, respectively. One of the followings is true. Choose the right one.
   a. $\hat{n}\times\hat{k}=0$ b. $\hat{n}\cdot\hat{k}=0$
   c. $\hat{n}\cdot\hat{k}\neq 0$ d. none of these

3. There exists an angle of incidence at which the reflected wave is completely extinguished. In order to get that condition, one of the following criteria should be fulfilled. Choose the correct one. ($n_1 \neq n_2$, symbols have their usual meanings)
   a. The polarization of the incident wave must be parallel to the plane of incidence.
   b. The polarization of the incident wave must be perpendicular to the plane of incidence.
   c. An arbitrarily polarized incident wave is sufficient to have the condition.
   d. There should not be any transmitted wave.

4. The attenuation of waves in conductors is due to the
   a. Real part of the wave number b. Imaginary part of the wave number
   c. Both real and imaginary parts of the wave number d. None of these.

5. In conductors,
   a. the magnetic field lags behind the electric field. b. the electric field lags behind the magnetic field.
   c. The electric and magnetic fields are in phase. d. none of these.

6. In the case of anomalous dispersion,
   a. refractive index increases with frequency b. refractive index decreases with frequency
   c. refractive index is independent of frequency d. none of these

7. Cauchy's formula reads as $n = 1 + A\left(1 + \frac{B}{\lambda^2}\right)$, where n is the refractive index and λ is the wavelength. A and B are respectively the
   a. coefficients of refraction and reflection
   b. coefficients of reflection and refraction
   c. coefficients of dispersion and reflection
   d. coefficients of refraction and dispersion

8. The boundary conditions at the inner wall of a wave guide are
   a. $\boldsymbol{E}^{\parallel} = 0, \boldsymbol{B}^{\parallel} = 0$
   b. $E^{\perp} = 0, B^{\perp} = 0$
   c. $E^{\perp} = 0, \boldsymbol{B}^{\parallel} = 0$
   d. $\boldsymbol{E}^{\parallel} = 0, B^{\perp} = 0$

9. A hollow wave guide does not admit
   a. TE waves
   b. TM waves
   c. TEM waves
   d. None of these

10. In rectangular waveguides, TE_{mn} mode cannot occur for
   a. $m = 0, n = 0$
   b. $m = 1, n = 0$
   c. $m = 0, n = 1$
   d. $m = 1, n = 1$

11. The electric field intensity $\vec{E}$ and magnetic field intensity $\vec{H}$ are coupled in free space in x and y directions respectively. The Poynting vector is given by
   a. $|\vec{E}||\vec{H}|\widehat{a_x}$
   b. $|\vec{E}||\vec{H}|\widehat{a_y}$
   c. $|\vec{E}||\vec{H}|\widehat{a_x}\widehat{a_y}$
   d. $|\vec{E}||\vec{H}|\widehat{a_z}$

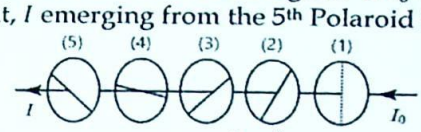
12. Which of the following is correct at a point on the interface between two different dielectric media, if there is no free charge or free current at the interface (symbols have their usual meaning)
   a. $E_1^{\perp} = E_2^{\perp}$
   b. $\frac{1}{\mu_1} B_1^{\perp} = \frac{1}{\mu_2} B_2^{\perp}$
   c. $E_1^{\parallel} = E_2^{\parallel}$
   d. $\mu_1 H_1^{\parallel} = \mu_2 H_2^{\parallel}$

13. Light is polarized to the maximum, when it is incident on a glass surface at an angle of incidence
   a. $tan^{-1}\left(\frac{2}{3}\right)$
   b. $tan^{-1}\left(\frac{3}{2}\right)$
   c. $tan^{-1}\left(\frac{1}{2}\right)$
   d. $tan^{-1}\left(\frac{1}{3}\right)$

14. The total energy stored in an electromagnetic field in a given volume in free space is
   a. $\frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$
   b. $\frac{1}{\sqrt{2}}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$
   c. $\frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0 c}\right)$
   d. $\frac{1}{2}(\varepsilon_0 E^2 + \mu_0 B^2)$

15. In double refraction, the vibrations of E ray are confined in
   a. Plane perpendicular to the principal section
   b. Plane parallel to the Principal section
   c. At an angle θ to the principal section
   d. Keeps on changing orientation

16. Five identical polaroids are placed coaxially with 45° angular separation between pass axes of adjacent polaroids as shown in the figure. If I_0 is intensity of unpolarized light, the intensity of light, I emerging from the 5th Polaroid is



a. $\frac{I_0}{4}$
b. $\frac{I_0}{8}$
c. $\frac{I_0}{16}$
d. $\frac{I_0}{32}$

17. The Nicol prism does not act as a polarizer when
    a. The angle of incidence is less than the critical angle for the O-ray
    b. The angle of incidence is greater than the critical angle for the O-ray
    c. The angle of incidence is less than the critical angle for the E-ray
    d. The angle of incidence is greater than the critical angle for the E-ray

18. In elliptically polarized light
    a. Amplitude of vibrations changes in magnitude and direction both
    b. Amplitude of vibrations changes only in magnitude
    c. Amplitude of vibrations changes only in direction
    d. No change in amplitude

19. What will be the velocity of an extraordinary ray in a given crystal with a refractive index of the extraordinary ray is 1.49 and the speed of light in vacuum is $3 \times 10^8\ m/s$?
    a. $3 \times 10^8\ m/s$
    b. $1.6 \times 10^8\ m/s$
    c. $1 \times 10^8\ m/s$
    d. $2 \times 10^8\ m/s$

20. If light is polarized by reflection, then the angle between reflected and refracted ray is
    a. π
    b. $\frac{\pi}{2}$
    c. 2π
    d. $\frac{\pi}{4}$

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. (i) Show that the standing wave $f(z,t) = A\sin(kz)\cos(kvt)$ satisfies the wave equation **3+2+2+3 =10**

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}.$$

(ii) Express the standing wave $f(z,t) = A\sin(kz)\cos(kvt)$ as the sum of a wave traveling to the left and a wave traveling to the right.

b. (i) Write down the differential forms of Maxwell's equations.
(ii) Show that the tangential components of electric field are continuous across the boundary between two dielectric media.

2. a. Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the $x - z$ direction. Give the explicit Cartesian components of $\vec{k}$ and $\hat{n}$. **5+5=10**

b. Deduce the boundary conditions governing the parallel components of the electric and magnetic field vectors when an EM wave passes through the interface between two dielectric media.

3. a.(i) A primitive model for an atom consists of a point nucleus $(+q)$ surrounded by a uniformly charged spherical cloud $(-q)$ of radius a. Calculate the atomic polarizability of such an atom. **3+3+4 =10**

(ii) If you take the model in 3a.(i) at face value, what natural frequency do you get? Put in the actual numbers. Where, in the electromagnetic spectrum, does this lie, assuming the radius of the atom is 0.5Å.

(iii) Find the coefficients of refraction and dispersion and compare them with those for hydrogen (H_2) at 0°C and atmospheric pressure: $A = 1.36 \times 10^{-4}, B = 7.7 \times 10^{-15}\text{m}^2$.

$\left(A = \frac{Nq^2}{2m\epsilon_0}\frac{f}{\omega_0^2}, B = \left(\frac{2\pi c}{\omega_0}\right)^2, N = \#\text{of molecules per unit volume} = \frac{N_A}{22.4\text{ liters}}, f = \#\text{ of electrons per molecule.}\right)$

Electromagnetic spectrum:

Region	Frequency (Hz)	Region	Frequency (Hz)
Radio	$< 3 \times 10^9$	Ultraviolet	$7.5 \times 10^{14} - 3 \times 10^{17}$
Microwave	$3 \times 10^9 - 3 \times 10^{12}$	X-Rays	$3 \times 10^{17} - 3 \times 10^{19}$
Infrared	$3 \times 10^{12} - 4.3 \times 10^{14}$	Gamma Rays	$> 3 \times 10^{19}$
Visible	$4.3 \times 10^{14} - 7.5 \times 10^{14}$		

4. a. Silver is an excellent conductor, but it's expensive. Suppose you were designing a microwave experiment to operate at a frequency of 10^{10} Hz. How thick would you make the silver coating? $[\mu \approx \mu_0, \sigma \approx 10^7(\Omega\text{m})^{-1}]$ **4+2+2+2 =10**

b. Show that the skin depth in a poor conductor ($\sigma \ll \epsilon\omega$) is $\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$ (independent of frequency).

c. Show that the skin depth in a good conductor ($\sigma \gg \epsilon\omega$) is $\lambda/2\pi$ (where λ is the wavelength in the conductor).

d. Show that in a good conductor the magnetic field lags the electric field by 45°.

$$\left[\tilde{k} = k_r + ik_i, k_r = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1\right]^{\frac{1}{2}}, \quad k_i = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]^{1/2}\right]$$

5. For the TE modes in rectangular wave guide, the wave number has the following expression **1+2+5+2 =10**

$$k = \sqrt{\left(\frac{\omega}{c}\right)^2 - \pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]} = \frac{1}{c}\sqrt{\omega^2 - \omega_{mn}^2}$$

a. The lowest cutoff frequency for a given wave guide occurs for which TE_{mn} mode? (Assume $a > b$)

b. Show that group velocity is less than the speed of light in free space.

c. Consider a rectangular wave guide with dimensions 2.28 cm × 1.01 cm. What TE modes will propagate in this wave guide, if the driving frequency is 1.7×10^{10} Hz?

d. Suppose you wanted to excite only one TE mode; what range of frequencies could you use?

6. **a.** Describe the working principle of a Nicol prism and how it acts as a polarizer and an analyzer. **7+3=10**

b. A beam of light AO is incident on a glass slab ($\mu = 1.54$) in a direction as shown in the figure below. The reflected ray OB is passed through a Nicol prism. On viewing through the prism, what is your observation regarding the intensity of light on rotating the prism?

7. **a.** What is the difference between left and right circularly polarized light? What happens if we add two left and right circular polarized waves? **4+4+2 =10**

b. An unpolarized light beam with an intensity of $I_0 = 16\,W/m^2$ is incident on a pair of polarizers. The first polarizer has its transmission axis aligned at 50° from the vertical. The second polarizer has its transmission axis aligned at 140° from the vertical. Calculate the intensity of the light

(i) when it emerges from the first polarizer.
(ii) when it emerges from the second polarizer.

c. Deduce the value of Brewster angle for a glass slab placed in air.

8. a. State and prove Poynting theorem for the flow of energy in an electromagnetic field. **8+2=10**

b. A radio station transmits a 10 *kW* signal at frequency of 10^7 *Hz*. Assume that it radiates at a point source. Calculate the amplitude of the electric and magnetic field strengths at a distance of 1 km from the point source.

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