

MASTER OF COMPUTER APPLICATION
First Semester
MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE
(MCA - 103)

Duration: 3Hrs.

Full Marks: 70

Part-A (Objective) =20
Part-B (Descriptive) =50

(PART-B: Descriptive)

Duration: 2 hrs. 40 mins.

Marks: 50

Answer any four from Question no. 2 to 8
Question no. 1 is compulsory.

1. (i) Show that $f: N \rightarrow N$, given by

$$f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x-1 & \text{if } x \text{ is even} \end{cases}$$

is both one-one and onto.

(ii) For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(a) $A + A'$ is a symmetric matrix.

(b) $A - A'$ is a skew-symmetric matrix.

(6+4=10)

2. (i) Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by

$R = \{(a, b): \text{both } a \text{ and } b \text{ are either odd or even}\}$. Show that R is an equivalence relation. Further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.

(ii) Determine whether each of the following relations are reflexive, symmetric and transitive:

(a) Relation R in the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as

$$R = \{(x, y) : 3x - y = 0\}$$

(b) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$$R = \{(x, y) : y \text{ is divisible by } x\} \quad (6+4=10)$$

3. (i) Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by

$$R = \{(x, y) : y = x + 1\}$$

(a) Depict this relation using an arrow diagram.

(b) Write down the domain, range and codomain of R .

(ii) Let \mathbb{N} be the set of natural numbers and E be the set of even numbers. Then find E^c .

(iii) Find $P(A)$ where $A = \{1, 2, 3, 4\}$.

(iv) Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$.

$$\text{Find } A', B', A' \cap B', A \cup B. \quad (2+2+3+3=10)$$

4. (i). Construct the truth table for the following statement formula

$$(p \wedge \sim q) \vee (q \wedge (\sim p \vee r))$$

(ii) Prove that $p \rightarrow q, q \rightarrow r \mid= p \rightarrow r$. (5+5=10)

5. (i) Translate each of the following sentences into a sentence into a statement formula:

(a) A necessary condition for x to be prime is that either x is odd or $x=2$.

(b) He is poor but honest.

(ii) State fundamental principle of counting.

(iii) (a) You can go from Delhi to Jaipur either by car or by bus or by train or by air, In how many ways can you plan your journey from Delhi to Jaipur and back to Delhi?

(b) Suppose you like variety and you don't want to return by the same mode of transport. How many different ways are possible? (4+2+4=10)

6. (i) How many 4 letter words, with or without meaning can be formed out of the letters of the word 'WONDER' if repetition of letters is not allowed?

(ii) In how many ways can final eleven be selected from 15 cricket players if

(a) There is no restriction

(b) One of them must be included

(c) One of them who is in bad form must always be excluded

(d) Two of them being leg spinners, one and only one leg spinners must be included? (2+8=10)

7. (i) Prove that in any group of n people there are at least two persons having same number friends. (It is assumed that if a person x is a friend of y then y is also a friend of x)

(ii) Find A^{-1} if $A = \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$ (5+5=10)

8. (i) What is an abelian group? Show that then set \mathbb{Q}^+ of all positive rational numbers forms an abelian group under the operation $*$ defined by

$$a * b = \frac{1}{2} ab \quad \forall a, b \in \mathbb{Q}^+.$$

(ii) Name some of the properties satisfied by an algebraic system. (7+3=10)

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(PART A - Objective Type)

I. Choose the correct answer:

1×20=20

1. A function $f : X \rightarrow Y$ is onto if and only if range of f is
(a) Y (b) X (c) $X - Y$ (d) $Y - X$
2. A function $f : X \rightarrow Y$ is bijective if it is
(a) One-one (b) onto (c) both one-one and onto (d) none
3. Let R be the relation in the set \mathbb{N} of natural numbers given by $R = \{(a, b) : a = b - 2, b > 6\}$.
Choose the correct answer
(a) (2,4) (b) (3,8) (c) (6,8) (d) (8,7)
4. If the set A has three elements and the set $B = \{3, 4, 5\}$, then the number of elements of the set $A \times B$ is
(a) 8 (b) 9 (c) 10 (d) 0
5. Let $A = \{2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$. Then
(a) $A \subseteq B$ (b) $A \subset B$ (c) $A \supset B$ (d) $A \not\subset B$
6. Let $A = \{a, b, c, d\}$ and $B = \{a, b\}$. Then $A \setminus B$ is
(a) $\{c, d\}$ (b) $\{a, b\}$ (c) $\{c\}$ (d) $\{d\}$
7. In mathematical logic we concern with the content of an argument. State **YES** or **NO**.
8. " \vdash " is known as
(a) Hasse symbol (b) Kleene's symbol
(c) Modus symbol (d) All of the above
9. A statement formula which is neither a tautology nor a contradiction is called
(a) Contingent (b) Tautology
(c) Contradiction (d) none
10. $n! = ?$
(a) $n \times (n-1) \times \dots \times 2 \times 1$ (c) n^2
(b) $(n-1) \times (n-2) \times \dots \times 2 \times 1$ (d) $\frac{n \times (n-1)}{2}$

11. $0! = ?$

- (a) 0 (b) 1 (c) 2 (d) 4

12. ${}^5P_3 = ?$

- (a) 59 (b) 69 (c) 60 (d) 61

13. ${}^nC_r = ?$

- (a) nC_n (b) rC_n (c) ${}^nC_{n-r}$ (d) rC_r

14. ${}^{12}C_5 = ?$

- (a) 792 (b) 790 (c) 795 (d) 699

15. Let $\{S, *, \oplus\}$ be an algebraic system. Then $a*(b \oplus c) = ?$

- (a) $a+(b*c)$ (b) $a+b*a+c$
(c) $a*b \oplus a*c$ (d) none

16. The identity element of the group $(\mathbb{N}, +)$ is

- (a) 0 (b) 1 (c) 2 (d) 4

17. In a group $(G, *)$, let $a, b \in G$, then $(a*b)^{-1} = ?$

- (a) $a*b$ (b) $a^{-1}*b^{-1}$ (c) $b^{-1}*a^{-1}$ (d) $b*a^{-1}$

18. A square matrix A is symmetric if

- (a) $A^T = A$ (b) $A^T = -A$
(c) all the diagonal entries are zero (d) none

19. Inverse of a square matrix, if exists, is not unique. State **TRUE** or **FALSE**.

20. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ be two matrices. Then

- (a) $AB = BA$ (b) $AB \neq BA$ (c) $AB = A+B$ (d) $AB = B-A$
