REV-01
MSM/01/05

2023/06

SET A

M.Sc. MATHEMATICS
SECOND SEMESTER
MECHANICS & TENSOR
MSM – 204 [REPEAT]

[USE OMR FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

(Objective)

Time : 30 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. If K.E of the system after explosion is greater than K.E of the system under subsequence motion,then it is known as
   a. Bertrand's Theorem
   b. Kelvin's Theorem
   c. Carnot's Theorem
   d. None of the above

2. Γ^{k}_{ij} is known as
   a. Christoffel's bracket of 1st Kind
   b. Christoffel's bracket of 2nd Kind
   c. Tensor
   d. Christoffel bracket

3. If $\bar{I}$ be the impulse of the particle from time t_1 to t_2 ,then
   a. $I = \int_{t_1}^{t_2} \bar{F}.dt$
   b. $\bar{I} = \int_{t_1}^{t_2} \bar{F}.dt$
   c. $\bar{I} = \int_{t_1}^{t_2} F dt$
   d. $\bar{I} = \int_{t_2}^{t_1} \bar{F}.dt$

4. Torque is denoted by
   a. $\bar{\Lambda} = \bar{r}.\bar{F}$
   b. $\bar{\Lambda} = \bar{r} \times m\bar{v}$
   c. $\bar{\Lambda} = \bar{r}.m\bar{v}$
   d. $\bar{\Lambda} = \bar{r} \times \bar{F}$

5. $A^{j}\delta^{i}_{j} = ?$
   a. A^{j}
   b. A^{i}
   c. δ^{i}_{j}
   d. δ^{j}_{i}

6. The equation of motion for the Lagrangian

a. $L = (1-\cos\theta)\ddot{\theta} - ag(1+\cos\theta)$

b. $L = (1+\cos\theta)\ddot{\theta} - ag(1+\cos\theta)$

c. $L = (1-\cos\theta)\ddot{\theta} + ag(1+\cos\theta)$

d. $L = (1-\cos\theta)\ddot{\theta} - ag(1-\cos\theta)$

7. If Impulse zero then

a. Momentum after Impulse is equal to momentum before Impulse

b. Momentum variable

c. Momentum Vanish

d. None of the above

8. $\dfrac{\partial A^i}{\partial x^j} + A^u \begin{Bmatrix} i \\ uj \end{Bmatrix} = ?$

a. $A^u{}_{;j}$

b. $A^u{}_{,j}$

c. $A^i{}_{,j}$

d. A^u

9. Hamiltonian H is identified as

a. Total energy of the System

b. Total K.E of the system

c. Total potential energy of the system

d. None of the above

10. In the term $a_{ij}x^j$, i is called

a. Dummy suffix

b. Real suffix

c. Kronecker delta

d. summation convention

11. One of the general equation of motion of a rigid body are

a. $\dfrac{d\vec{\Omega}}{dt} = \sum \vec{r^j} \times \vec{F}$

b. $\dfrac{d\Omega}{dt} = \sum \vec{r^j} \times \vec{F}$

c. $\dfrac{d\vec{\Omega}}{dt} = \sum \vec{r^j} \times \vec{V}$

d. None of the above

12. The Equation of motion of a pendulum is

a. $\ddot{\theta} = \dfrac{g}{l^2}\theta$

b. $\ddot{\theta} = -\dfrac{g}{l}\theta$

c. $\ddot{\theta} = \dfrac{g}{l}\theta^2$

d. $\ddot{\theta} = \dfrac{g}{l}\theta$

13. A^{ij} are called component of contravariant tensor if

a. $\bar{A}^{ij} = \frac{\partial \bar{x}^i}{\partial x^r}\frac{\partial \bar{x}^j}{\partial x^s}A^{rs}$

b. $\bar{A}^{ij} = \frac{\partial \bar{x}^j}{\partial x^r}\frac{\partial \bar{x}^i}{\partial x^s}A^{rs}$

c. $\bar{A}_{ij} = \frac{\partial \bar{x}^j}{\partial x^r}\frac{\partial \bar{x}^i}{\partial x^s}A_{rs}$

d. $\bar{A}_{ij} = \frac{\partial \bar{x}^i}{\partial x^r}\frac{\partial \bar{x}^j}{\partial x^s}A_{rs}$

14. The equation of motion of a particle in a plane under inverse squre law are

a. $r^2\dot{\theta} = c$ and $\ddot{r} - r\dot{\theta}^2 = \frac{\mu}{r^2}$

b. $-r^2\dot{\theta} = c$ and $\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2}$

c. $r^2\dot{\theta} = c$ and $\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2}$

d. None of the above

15. Which of the following is correct

a. $\frac{d\Omega}{dt} = \Lambda$

b. $\frac{d\bar{\Omega}}{dt} = \bar{\Lambda}$

c. $\frac{d\Omega}{dt} \neq \Lambda$

d. $\frac{d\Omega}{dt} < \Lambda$

16. Cyclic coordinate denoted as

a. $\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) = 0$

b. $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0$

c. $\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0$

d. $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) \neq 0$

17. If T' denote K.E of the system just after explosion and T denote K.E of the system before explosion,then

a. $T' > T$

b. $T' < T$

c. $T' \leq T$

d. $T' \geq T$

18. If $\overline{\Lambda}$ be the torque applied on a particle from time t_1 to t_2, then the time integral $\bar{\tau} = \int_{t_1}^{t_2} \overline{\Lambda} dt$ is known as

a. Impulse
b. Angular Impulse
c. Angular Momentum
d. Momentum

19. The Equation of motion under impulsive force of a system which include two colliding bodies with m_α be the mass, r_α be the position vector and q_α be the velocity is

a. $\sum m_\alpha (\bar{q}'_\alpha - \bar{q}_\alpha)\partial \bar{r}'_\alpha = \sum \bar{I}_\alpha . \partial \bar{r}_\alpha$

b. $\sum m_\alpha (\bar{q}'_\alpha - \bar{q}_\alpha)\partial \bar{r}_\alpha = \sum I_\alpha . \partial \bar{r}_\alpha$

c. $\sum m_\alpha \times (\bar{q}'_\alpha - \bar{q}_\alpha)\partial \bar{r}_\alpha = \sum I_\alpha . \partial \bar{r}_\alpha$

d. None of the above

20. The Christoffel bracket of 1st kind is

a. $(ij,k) = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right)$

b. $(ij,k) = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial x^{ji}} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right)$

c. $(ji,k) = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right)$

d. $(ji,k) = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right)$

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Descriptive

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. What do you mean by Torque? Find the general equation of motion of a rigid body under impulsive forces? 2+8=10

2. Prove that the Outer Product of two vectors is a tensor of order two. What is the rank of Kronecker delta and prove it. 6+4=10

3. Write the Lagrange's equation for a conservative holonomic system. Find the Lagrange equation for holonomic system. 2+8=10

4. What is Hamiltonian Principle? A particle of mass m moves in a force field of potential V. Write the Hamiltonian and the Hamiltonian equation in spherical polar coordinate form. 3+7=10

5. Prove that 5+5=10

   (a) $[ij,h] = g_{kh}\begin{Bmatrix} k \\ ij \end{Bmatrix}$

   (b) $[ij,k]+[jk,i] = \frac{\partial g_{ik}}{\partial x^j}$

6. Prove that Impulse= Change in Momentum? The velocities of a particle along and perpendicular to the radious vector are λr and $\mu\theta$. Find the path and show that the acceleration along and perpendicular to the radious vector are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta(\lambda + \frac{\mu}{r})$. 3+7=10

7. Write the definition of Displacement of a vector, Angular velocity, Angular momentum, Bertrand's Statement and Christoffel Bracket of 1st Kind. 2×5=10

8. What is Kelvin's Theorem? Prove that Kinetic energy of the system after collision is less than the Kinetic energy of a system before collision. 1+9=10

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