REV-01
MSM/01/05

2023/06

SET A

M.Sc. MATHEMATICS
SECOND SEMESTER (REPEAT)
DIFFERENTIAL EQUATION-II
MSM-202
[USE OMR SHEET FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 30 mins. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. $f(x)=|y|$ satisfies Lipschitz's condition.
   a. True
   b. False
   c. Undetermined
   d. Under some conditions

2. $T_n(x)=$
   a. $Cos(nCos^{-1}x)$
   b. $Cos(mCos^{-1}x)$
   c. Both a and b are correct
   d. None

3. If $U_n(-1)$
   a. 0
   b. 1
   c. -1
   d. None

4. $T_{2n+1}(0)=$
   a. 0
   b. 1
   c. 1
   d. None

5. $f(x)=e^x$ is an:
   a. Even function
   b. Odd function
   c. Cannot be determined
   d. None

6. $f(x)=Sinx$ is a periodic function with period:
   a. π
   b. 2π
   c. 0
   d. None

7. $\beta(m,n)=$
   a. $\beta(n,m)$
   b. Γm
   c. Γn
   d. All are correct

8. $\Gamma(-\frac{1}{2})=$
   a. $\Gamma\pi$
   b. $-\Gamma\pi$
   c. Cannot be calculated
   d. 0

9. $T_6(x) =$

a. 0 | b. 1
c. Impossible to calculate | d. None

10. $\tan x$ and $\cot x$ both have the:

a. Same period | b. Different period
c. No period | d. None

11. If $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ then given system of differential equation are:

a. Compatible | b. Non-compatible
c. We cannot say | d. None

12. $\frac{\partial z}{\partial x} = 5x + 4y$ and $\frac{\partial z}{\partial y} = 6x - 7y$ have:

a. Common solution | b. No common solution
c. No solution | d. None

13. If $p = P(x, y)$ and $q = Q(x, y)$ is a compatible system of equations then their general solution is:

a. $dz = pdx - qdy$ | b. $dz = pdx + qdy$
c. Both a and b are correct | d. None

14. If $y_1(x)$ and $y_2(x)$ are two solutions of a 2nd order linear differential equation, then their linear combination:

a. Is also a solution of the same diff equn | b. Cannot be a solution of the same diff equn
c. Both a and b are possible | d. None

15. $\Gamma(-ve\ value)$ is:

a. Possible | b. Not possible
c. Possible under certain condition | d. None

16. $T_3(x) =$

a. 4x-3 | b. 3x-2
c. 2x+5 | d. None

17. $U_2(x) =$

a. $2x(1-x^2)^{1/2}$ | b. 1
c. 3x | d. None

18. $p = x^2 - ay$ and $q = y^2 - ax$ are:

a. Compatible | b. Not compatible
c. Cant not be determined | d. None

19. If $f(x+T)=f(x)$ then period is:

a. x

b. T

c. Both a and b

d. None

20. $\beta(5,10)=$

a. 15150

b. 15151

c. 15152

d. None

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(PART-B : Descriptive)

Time : 2 hr. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. Solve the following one dimensional wave equation using variable separable method 10

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

2. a. Show that $\phi_1(x) = e^{2x}, \phi_2(x) = xe^{2x}$ and $\phi_3(x) = x^2 e^{2x}$ are linearly independent solutions of $y''' - 6y'' - 8y = 0$ on the interval $0 \le x \le 1$ 5+5=10

   b. Check whether the differential equations $\frac{\partial z}{\partial x} = 5x - 7y$ and $\frac{\partial z}{\partial y} = 6x + 8y$ are compatible or not.

3. a. Illustrate by an example that a continuous function may not satisfy Lipscitz condition in a rectangle. 5+5=10

   b. Find Fourier series of the function $f(x) = xSinx$, $0 \le x \le 2\pi$

4. a. Show that $(1 - x^2)^{1/2} T_n(x) = U_{n+1}(x) - xU_n(x)$ 5+5=10

   b. Prove that the continuity of $f(x, y)$ is not enough to guarantee the uniqueness of the solution of the initial value problem

$$\frac{dy}{dx} = f(x, y) = \sqrt{|y|}\ , y(0) = 0$$

5. a. Show that $T_n(x)$ *and* $U_n(x)$ are the independent solutions of the differential equation $(1 - x^2)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} + n^2 y = 0$ 5+5=10

   b. Prove that $\beta(m, n) = \frac{\Gamma m\, \Gamma n}{\Gamma(m + n)}$

6. a. Solve the following system of equations: 5+5=10

$$\frac{dx}{dt} = S\text{int} - y$$

$$\frac{dy}{dt} = \cos t - x$$

b. Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

7. 5+5=10

a. Show that $\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}}dx = \begin{cases} 0, & m \neq n \\ \pi/2, & m = m \neq 0 \\ \pi, & m = n = 0 \end{cases}$

b. What are the Euler's formula and Dirichlet's conditions for Fourier Series?

8. a. Use separation of variable method to solve 5+5=10

$U_t = U_x + U$ with $U(x,0) = 6e^{-3x}$

b. State and prove Sturmm Liouville theorem.

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