REV-01
MSM/24/29

2023/06

SET A

M.Sc. MATHEMATICS
SECOND SEMESTER
COMPLEX ANALYSIS
MSM – 201
[USE OMR FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

(Objective)

Time : 30 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. The residue of $\dfrac{1-e^{2z}}{z^4}$ is
   a. $\dfrac{-4}{3}$
   b. 1
   c. 3
   d. $e^{-i\frac{\pi}{2}}$

2. The principal value of $Log(i^{1/4})$ is
   a. $i\pi$
   b. $\dfrac{i\pi}{2}$
   c. $\dfrac{i\pi}{4}$
   d. $\dfrac{i\pi}{8}$

3. Consider the function $f(x)=x^2+iy^2$ and $g(x)=x^2+y^2+ixy$ at $z=0$ then
   a. f is analytic but not g
   b. g is analytic but not f
   c. Both the functions are analytic
   d. None is analytic

4. For $z \in C$ define $f(z)=\dfrac{e^z}{e^z-1}$, then
   a. The only singularities of f are poles
   b. f has infinitely many poles in the imaginary axis
   c. Each pole of f is simple
   d. All of the above

5. The function $f(z)=|z|$ is
   a. differentiable
   b. Nowhere differentiable
   c. Differentiable at $z=0$
   d. None

6. F is said to be entire if

a. F is analytic on C
b. F is regular on C
c. ∞ is the only possible singularity
d. All of the above

7. An isolated singularity which is neither a removable singularity nor a pole is called

a. Essential singularity
b. Essential removable singularity
c. Isolated removable singularity
d. None

8. The number $\sqrt{2}\ e^{i\pi}$ is

a. Rational Number
b. A transcendental number
c. An irrational number
d. An imaginary number

9. A conformal transformation $u = u(x,y), v = v(x,y)$ maps the curves C_1 and C_2 intersecting at $z_0 = (x_0, y_0)$ in xy plane respectively into the curves C_1' and C_2' intersecting at $w_0 = (u_0, v_0)$ in uv plane. The corresponding angles between the curves in the two planes at z_0 and w_0 are

a. equal in magnitude but does not preserve the sense
b. not necessarily equal in magnitude but preserve the sense
c. equal in magnitude and preserve the sense
d. None of the above

10. The translation is a transformation from a z-plane to w-plane generally given by: (α is a given complex number)

a. $w = f(z) = \frac{1}{z}$
b. $w = z + \alpha$
c. $w = \frac{1}{z+\alpha}$
d. $w = z^2 + \alpha$

11. The transformation $w = e^{i\theta}z$ is

a. simply a translation.
b. a combination of translation and rotation
c. simply a rotation.
d. combination of stretching and rotation

12. The translation $w = f(z) = \alpha z$, where α is a given complex number is

a. a combination of rotation and stretching
b. a combination of rotation and translation
c. a combination of translation and inversion
d. simply a stretching

13. The Jacobian of the transformation $w = f(z) = (1+i)z + 1 - i$ is

a. 1
b. 2
c. 4
d. ½

14. The value of the integral $\int_C \frac{1}{z-1}\,dz, C:|z| = 4$ is equal to

a. πi
b. $2\pi i$
c. $4\pi i$
d. 0

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15. In the Laurentz series of $\frac{Sin\, z}{z^2}$ at $z = 0$ the co-efficient of $\frac{1}{z}$ is

a. zero
b. π
c. -1
d. 1

16. $f(z) = \frac{Sinz}{z^2}$ has

a. A removable singularity at $z = 0$
b. A pole of order 1 at $z = 0$
c. No singularities
d. An essential singularity at $z = 0$

17. The residue of $e^{1/z}$ at $z = 0$ is

a. 2
b. ∞
c. 1
d. -2

18. The residue of $\frac{Sinz}{z^8}$ at $z = 0$ is

a. 0
b. $\frac{1}{7!}$
c. $-\frac{1}{7!}$
d. None

19. $\int_0^{1+i}(x^2 - iy)dz$ along $y = x$ is

a. $-\frac{1}{6}(5-i)$
b. $\frac{1}{6}(5-i)$
c. $-\frac{1}{6}(5+i)$
d. $\frac{1}{6}(5+i)$

20. $\oint_C \frac{z^2+5}{z-3}dz$ where C is the circle $|z| = 1$

a. 0
b. $2\pi i$
c. $-2\pi i$
d. πi

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(Descriptive)

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. **a.** Determine the analytic function whose real part is $e^{2x}(xCos2y - ySin2y)$ 5+5=10

b. Determine the analytic function whose imaginary part is $Sinhx\ Coshy$

2. **a.** Prove that the angle between two curves C_1 and C_2 passing through the point z_0 in the z-plane is preserved in magnitude and sense under the transformation $w = f(z)$ if $f(z)$ is analytic at z_0 and $f'(z_0) = 0$. 5+5=10

b. Find the Jacobian of the transformation $w = \sqrt{2}\, e^{i\frac{\pi}{4}} z + (1 - 2i)$.

3. **a.** If $w = f(z) = u + iv$ is analytic in a region R prove that the Jacobian of the transformation is given by 5+5=10

$$J = \frac{\partial(u,v)}{\partial(x,y)} = |f'(z)|^2$$

b. Determine the region of w-plane into which the 1st quadrant of the z-plane is mapped by the transformation $w = z^2$.

4. 5+5=10

a. Calculate the integral $\int_0^{1+i} (x - y + ix^2)dz$ along the real axis from $z = 0$ to $z = 1$ and the along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$

b. Evaluate $\frac{1}{2\pi i}\oint_C \frac{ze^z}{(z-a)^3}dz$ where C is the circle $|z| = a$

5\. a. Evaluate $\oint_C \frac{Sin^6 z}{(z-\frac{\pi}{6})^3} dz$ where C is the circle $|z|=1$ 5+5=10

b. State and prove Morera's theorem.

6\. a. Show that the function $e^{-2xy}.Sin(x^2-y^2)$ is harmonic 5+5=10

b. Apply Residue theorem to evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)} dz$ where

C is the circle $|z|=\frac{3}{2}$

7\. a. Apply Residue theorem to evaluate $\oint_C \frac{2z^2+5}{(z+2)^2(z^2+4)} dz$ on the 5+5=10

circle $C:|z|=3$

b. Expand by Laurentz series $\frac{1}{z^2-4z+3}$ for the region $1<|z|<3$

8\. a. Evaluate the following integral by using Residue theorem 5+5=10

$\oint_C \frac{z}{(z-1)(z-2)^2} dz$ where C is the circle $|z-2|=\frac{1}{2}$

b. Expand by Laurentz series $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region $|z|>4$

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