2023/06

SET A

M.Sc. MATHEMATICS
THIRD SEMESTER
NUMBER THEORY
MSM - 301[SPECIAL REPEAT]
[USE OMR FOR OBJECTIVE PART]

Duration : 3 hrs. Full Marks : 70

(Objective)

Time : 30 min. Marks : 20

Choose the correct answer from the following: *1X20=20*

1. For a nonzero integer a, $\gcd(a,0)$ is:
   a. a
   b. a
   c. $|a|$
   d. None of these

2. $2^{35}-1$ is divisible by
   a. 31
   b. 129
   c. Both 31 & 129
   d. None of these

3. Suppose $m,r>0$ and $\{a_1,a_2,\cdots\cdots,a_t\}$ is a CRS modulo m then $\{a_1+r,a_2+r,\cdots\cdots,a_t+r\}$ is a
   a. CRS modulo m iff $(r,m)=1$
   b. RRS modulo m iff $(r,m)=1$
   c. RRS modulo m
   d. CRS modulo m

4. Which of the following is/are true
   a. $a^{21}\equiv a\ (mod\ 5)$
   b. $a^{21}\equiv 1\ (mod\ 5)$
   c. $a^{20}\equiv a\ (mod\ 5)$
   d. None of these

5. If $a^{\frac{p-1}{2}}\equiv 1\ (\text{mod}\ p)$, where p is an odd prime, then
   a. a is a quadratic residue of p
   b. a is a quadratic non-residue of p
   c. a is a primitive root of p
   d. None of these

6. Which of the following primes satisfy the congruence $a^{24}\equiv 6a+2\ (\text{mod}\ 13)$?
   a. 41
   b. 47
   c. 68
   d. 83

7. The linear congruence $99x\equiv 1\ (\text{mod}\ 131)$ has
   a. No solution
   b. Unique solution
   c. Finite number of solution
   d. None of these

8. If p is an odd prime, then $1^{167}+2^{167}+3^{167}+\cdots+166^{167}$ mod 167 is
   a. 1
   b. -1
   c. 0
   d. None of these.

9. The last two digits of 7^{81} are
   a. 47
   b. 17
   c. 37
   d. 7

10. Consider the congruence $x^n \equiv 2 \mod 13$. This congruence has a solution if

a. $n = 4$ b. $n = 5$
c. $n = 6$ d. None of these

11. If a is a primitive root of modulo n, then

a. $n = 15$ b. $n = 20$
c. $n = 25$ d. None of these

12. The value of $\left(\frac{16}{31}\right)$ is:

a. 0 b. 1
c. -1 d. None of these

13. If p is an odd prime, then $\sum_{a=1}^{p-1} \left(\frac{a}{p}\right)$ is

a. 0 b. 1
c. -1 d. None of these

14. If 2 is primitive root of 13 and a is a quadratic residue of 13, then

a. $a = 8$ b. $a = 6$
c. $a = 7$ d. $a = 12$

15. $\left(\frac{p}{q}\right) = \left(\frac{q}{p}\right)$ if

a. $p \equiv 1 (\text{mod } 4)$ and $q \equiv 1 (\text{mod } 4)$ b. $p \equiv 3 (\text{mod } 4)$ or $q \equiv 3 (\text{mod } 4)$
c. $p \equiv 1 (\text{mod } 4)$ or $q \equiv 1 (\text{mod } 4)$ d. $p \equiv 3 (\text{mod } 4)$ and $q \equiv 3 (\text{mod } 4)$

16. For the Fibonacci sequence, (u_n, u_{n+1}) is

a. 1, for every $n \neq 1$ b. 1, for every $n > 1$
c. 1, for every $n \geq 1$ d. None of these

17. The value of C_2 is

a. $\frac{1}{2}$ b. $\frac{1}{3}$
c. $\frac{3}{8}$ d. $\frac{3}{2}$

18. The value (u_m, u_n) is:

a. u_d where $d = \gcd(m, n)$ b. u_d where $d = \text{lcm}(m, n)$
c. u_d where $d = \frac{n}{m}$ d. u_d where $d = \frac{m}{n}$

19. Given a natural number $n > 1$ such that $(n-1)! \equiv -1 (\text{mod } n)$ Then

a. $n = p^k$ where p is prime, $k > 1$. b. $n = pq$ where p, q are primes.
c. $n = p^2 q$ where p, q and r is primes. d. $n = p$ where p is prime.

20. Which of the following statement is **false**?

There exists an integer x such that

a. $x \equiv 23 (\text{mod } 1000), x \equiv 45 (\text{mod } 6789)$ b. $x \equiv 32 (\text{mod } 1000), x \equiv 44 (\text{mod } 9876)$

c. $x \equiv 23 (\text{mod } 1000), x \equiv 54 (\text{mod } 6789)$ d. $x \equiv 32 (\text{mod } 1000), x \equiv 45 (\text{mod } 9876)$

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Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Prove that - Given two non-zero integers a and b, there exist integers x and y such that $(a,b) = ax + by$. **5+5=10**
   b. Find the value of x and y to satisfy $423x + 198y = 9$.

2. a. Prove that – If the integer a have order k modulo n then $h > 0$ then a^h has order $\frac{k}{(h,k)}$ modulo n. **4+3+3 =10**

   b. If p and q are distinct primes, prove that

   $$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$$

   c. Find the remainder of $111^{333} + 333^{111}$ are divided by 7.

3. a. Find the common solution of the following system of linear congruence: **5+5=10**

   $$x \equiv 5\ (mod\ 6)$$
   $$x \equiv 4\ (mod\ 11)$$
   $$x \equiv 3(mod\ 17)$$

   b. Solve the following linear congruence:

   $$25x \equiv 10 \pmod{29}$$

4. a. State and prove Euler's criterion. **5+5=10**
   b. Prove that - $[a,b](a,b) = ab$.

5. a. Find the solution of $3x^2 + 9x + 7 \equiv 0 (\text{mod } 13)$. **5+5=10**
   b. Determine whether the following congruence has solution or not:

   $$x^2 \equiv -46 \pmod{17}$$

6. a. Prove that – There is an infinite number of primes. **4+3+3 =10**
   b. Let p be a prime number. Then prove that $x^2 \equiv 1 \pmod{p}$ if and only if $x \equiv \pm 1 (\text{mod } p)$.
   c. Find the remainder of $4(29!) + 5!$ divided by 31.

7. a. Prove that – If $r_1, r_2, \cdots, r_n$ is a CRS modulo$n$, then $ar_1 + b, ar_2 + b, \cdots, ar_n + b$ is also CRS mod n, where $(a, n) = 1$ & b is an integer. 4+3+3 =10

b. Find the value of $\left(\frac{29}{53}\right)$.

c. Express the rational number $\frac{187}{57}$ as finite simple continued fraction.

8. a. Prove that – For the Fibonacci sequence, $(u_n, u_{n+1}) = 1$ for every $n \geq 1$. 5+5=10

b. Prove that – The kth convergent of the simple continued fraction $[a_0; a_1, \cdots, a_n]$ has the value $C_k = \frac{p_k}{q_k}$ $0 \leq k \leq n$.

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