2023/06

SET A

M.Sc. MATHEMATICS
SECOND SEMESTER
DIFFERENTIAL EQUATION I
MSM – 205
[USE OMR FOR OBJECTIVE PART]

Duration: 1.30hrs **Full Marks: 35**

(Objective)

Time: 15 min. **Marks: 10**

Choose the correct answer from the following: ***1×10=10***

1. If the partial differential equation is in the form of $p^2 + q^2 = 1$ then the PDE is in the form of
   a. Linear
   b. Semi linear
   c. Quasi linear equation
   d. Non linear

2. The relation $z = (x + a)(y + b)$ represent the PDE is
   a. $z = \frac{p}{q}$
   b. $z = pq$
   c. $z = p - q$
   d. None of these

3. Solution of the PDE $\frac{\delta Z}{\delta x} + 4Z = \frac{\delta Z}{\delta t}$, given $z(x, 0) = 4e^{-3x}$ is
   a. $z = 4e^{3x+1}$
   b. $z = 3e^{4x+1}$
   c. $z = e^{-3x+1}$
   d. $z = 4e^{-3x+1}$

4. The general solution of the PDE $2\frac{\delta^2 z}{\delta x^2} + 5\frac{\delta^4 z}{\delta x \delta y} + 2\frac{\delta^2 z}{\delta y^2} = 0$ is
   a. $z = \varphi(2y - x)$
   b. $z = \psi(y - 2x)$
   c. $z = \varphi(2y - x) + \psi(y - 2x)$
   d. $z = \varphi(y - x) + \psi(y + x)$

5. If the partial differential equation is in the form of p + q = z + xy then the equation is
   a. Linear
   b. Semi linear
   c. Quasi linear equation
   d. Non linear

6. The general solution of the PDE $\frac{\delta^2 z}{\delta x^2} - 5\frac{\delta^4 z}{\delta x \delta y} + 4\frac{\delta^2 z}{\delta y^2} = \sin(4x + y)$ is
   a. $z = \frac{1}{3}x\cos(4x + y)$
   b. $z = f_1(y + x) + f_2(y + 4x)$
   c. $z = f(y + x) - \frac{1}{3}x\cos(4x + y)$
   d. $z = f_1(y + x) + f_2(y + 4x) - \frac{1}{3}x\cos(4x + y)$

7. Equation $\frac{\delta^2 z}{\delta x^2} - 2\left(\frac{\delta^2 z}{\delta x \delta y}\right) + \left(\frac{\delta z}{\delta y}\right)^2 = 0$ is of order

a. 1
b. 2
c. 3
d. None of these

8. If the partial differential equation is in the form of $(2x + 3y)p + 4xq - 8pq = x + y$ then the equation is

a. Linear
b. Semi linear
c. Quasi linear equation
d. Non linear

9. The relation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ represents the partial differential equation is

a. $z = p + q$
b. $z = p - q$
c. $2z = xp + yq$
d. $2z = \frac{xp}{yq}$

10. The PDE formed by eliminating arbitrary functions from the equation $z = f(x^2 - y^2)$ is

a. $xp + yq = 0$
b. $xq + yp = 0$
c. $\frac{x}{y} = p$
d. $\frac{x}{y} = q$

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Descriptive

Time : 1 hr. 15 mins. **Marks : 25**

[Answer question no.1 & any two (2) from the rest]

1. Find the equation of the integral surface of the differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$, Which contains the line $x + y = 0, z = 1$ 5

2. a. Find the equation of the integral surface of the differential equation $2y(z - 3)p + y(2x - z)q = (2x - 3)y$, Which passes through the circle $z = 0,\ x^2 + y^2 = 2x$ 5+5=10

   b. Find the complete integral of the equation px + qy = pq

3. a. Solve $(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$ 5+5=10

   b. Derive the two dimensional Heat equation

4. a. Solve $r + (a + b)s + abt = xy$ by Monge's Method 5+5=10

   b. Derive the two dimensional wave equation

5. a. Show that the functional $I_1[y(x)] = \int_a^b \{y'(x) + y(x)\}\,dx$ is linear in the class $C^1[a,b]$ but the functional $I_2[y(x)] = \int_a^b [p(x)\{y^1(x)\}^2 + q(x)\{y(x)\}^2]\,dx$ is nonlinear. 5+5=10

   b. Let a functional $I[y(x)]$ define on the class $C^1[0,1]$ be given by $I[y(x)] = \int_0^1 [1 + \{y^1(x)\}^2]^{\frac{1}{2}}\,dx$ then prove that $I[1] = 1,\ \ I[x] = \sqrt{2}, and\ I[x^2] = \frac{\sqrt{5}}{2} + (1/4)sinh^{-1}2$

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