M.Sc. PHYSICS
SECOND SEMESTER
MATHEMATICAL PHYSICS
MSP - 201
[USE OMR FOR OBJECTIVE PART]

SET A

Duration: 3 hrs.
Full Marks: 70

(Objective)

Time: 30 min.
Marks: 20

Choose the correct answer from the following:
$1 X 20 = 20$

1. The differential equation $x^2 y''(x) + x\, y'(x) + (x^2 - n^2)\, y(x) = 0$ is called
   a. Legendre Equation
   b. Hermite Equation
   c. Bessel Equation
   d. Laguerre Equation

2. The value of kroneker delta symbol δ_i^i in 3-dimensional space is
   a. 1
   b. 3
   c. 6
   d. none of the above

3. The expression of $J_0(x)$ is
   a. $1 - \frac{1}{(1!)^2}\left(\frac{x}{2}\right)^2 + \frac{1}{(2!)^2}\left(\frac{x}{2}\right)^4 - \frac{1}{(3!)^2}\left(\frac{x}{2}\right)^6 + \cdots$
   b. $1 - \frac{1}{(2!)^2}\left(\frac{x}{2}\right)^2 + \frac{1}{(3!)^2}\left(\frac{x}{2}\right)^4 - \frac{1}{(4!)^2}\left(\frac{x}{2}\right)^6 + \cdots$
   c. $1 - \left(\frac{x}{2}\right)^2 + \frac{1}{2^2}\left(\frac{x}{2}\right)^4 - \frac{1}{3^2}\left(\frac{x}{2}\right)^6 + \cdots$
   d. $1 - \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^4 - \left(\frac{x}{2}\right)^6 + \cdots$

4. The outer product of two mixed tensors $A_\gamma^{\alpha\beta}$ and B_α^ρ of rank 3 and 2 respectively followed by a contraction produces a new tensor of rank
   (a) 0
   (b) 1
   (c) 3
   (d) 5
   a. 0
   b. 1
   c. 3
   d. 5

5. The expression $J_{-n}(x)$ is
   a. $(-1)^{n-1} J_n(x)$
   b. $(-1)^n J_n(x)$
   c. $(-1)^{n+1} J_n(x)$
   d. $-J_n(x)$

6. The number of independent components of a symmetric tensor of rank 2 in n-dimensional space is
   a. n^2
   b. $\frac{n(n+1)}{2}$
   c. $\frac{n+1}{2}$
   d. $\frac{n(n-1)}{2}$

7. Which one of the following recurrence relation is true
   a. $2 J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$
   b. $2 J'_n(x) = -J_{n-1}(x) - J_{n+1}(x)$
   c. $2 J'_n(x) = J_{n-1}(x) + J_{n+1}(x)$
   d. $2 J'_n(x) = -J_{n-1}(x) + J_{n+1}(x)$

8. The value of $g_{\mu\nu} A^{\mu\lambda}$ is ($A^{\mu\lambda}$ is a contravariant tensor of rank 2 and $g_{\mu\nu}$ is the covariant metric tensor of rank 2)
   a. $A^{\nu\lambda}$
   b. $A_{\lambda\nu}$
   c. A_ν^λ
   d. A_ν^μ

[1]

9. $\left(\frac{1}{1-t}\right) e^{-\left(\frac{t}{1-t}\right)x}$ is the generating function of
   a. Hermite polynomial
   b. Laguerre polnomial
   c. Bessel polynomial
   d. Legendre polynomial

10. If $(G,\cdot)$ is a group such that $(ab)^{-1} = a^{-1}b^{-1}$ for $\forall a, b \in G$, then G is a/an
    a. commutative subgroup
    b. Abelian group
    c. Non-Abelian group
    d. none of these

11. The value of $L_1\left(\frac{1}{2}\right)$ will be
    a. $-\frac{1}{2}$
    b. 0
    c. 1
    d. $\frac{1}{2}$

12. This is an abelian group $\{-3n : n \varepsilon Z\}$ under?
    a. division
    b. subtraction
    c. addition
    d. multiplication

13. If $f(x) = 2 + x = \sum_{n=0}^{1} c_n\, p_n\,(x)$ then the value of c_0 is
    a. 1
    b. 2
    c. 0
    d. -1

14. If K is kernel of a group homomorphism f : G → H, then which statement is not true?
    a. K is an abelian subgroup of G
    b. K is a normal subgroup of G
    c. K = {e} for some homomorphism
    d. K = G for some homomorphism

15. The value of $P_2^1 (\sin\theta)$ at $\theta = \frac{\pi}{4}$ will be
    a. $\frac{1}{2}$
    b. $\frac{3}{2}$
    c. $\frac{3}{4}$
    d. $\frac{1}{4}$

16. If b and c are elements in a group G and if $b^5 = c^3 = e$, where e is the identity of G, then the inverse of $b^2cb^4c^2$ must be
    a. $cb^2c^2b^4$
    b. $c^2b^4cb^2$
    c. cbc^2b^3
    d. $b^4c^2b^2c$

17. The value of $\int_{-\infty}^{\infty} e^{-x^2} [H_1]^2 dx$ will be
    a. $\sqrt{\pi}$
    b. $\sqrt{\frac{\pi}{2}}$
    c. $\sqrt{\pi}/2$
    d. $2\sqrt{\pi}$

18. For every group G, the identity mapping I_G defined by $I_G : G \longrightarrow G$, $I_G(x) = x$, $\forall x \in G$ is a/an
    a. homomorphism of G onto itself
    b. isomorphism of G onto itself
    c. one-one mapping
    d. none of these

19. If $f(x) = 1 + 2x = \sum_{n=0}^{1} c_n H_n(x)$ then the value of c_1 is
    a. 1
    b. 2
    c. ½
    d. 0

20. A subset H of a group $(G, *)$ is group if
    a. $a, b \in H \Rightarrow a * b \in H$
    b. $a \in H \Rightarrow a^{-1} \in H$
    c. $a, b \in H \Rightarrow a * b^{-1} \in H$
    d. H contains the identity element

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(Descriptive)

Time : 2 hrs. 30 mins.   Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Obtain a general of order n of the Legendre second-order   8+2=10
       differential equation.
    b. Define Legendre polynomial $p_n(x)$ in differential form

2. a. Define Christofell's symbols of first and second kind. Show that,   4+4+2
       $\Gamma_{\mu\nu}^{\sigma} = g^{\sigma\lambda}\Gamma_{\lambda,\mu\nu}$.   =10
    b. Prove that the addition and subtraction of two mixed tensors of
       rank 2 each produces a new mixed tensor of rank 2.
    c. If $g_{\mu\nu} = 0$ for $\mu \neq \nu$ and μ, ν, σ are unequal indices, find the value
       of $\Gamma_{\mu\mu}^{\nu}$.

3. a. Using the Rodrigue's formula of Hermite polynomial obtain an   4+4+2
       expression of $H_3(x)$ =10
    b. Prove that the Hermite polynomial satisfies the recurrence
       relation $2nH_{n-1}(x) = H_n'(x)$.
    c. Find an expression of $H_{2n}(x)$ at $x = 0$.

4. a. Show that the additive group $(R, +)$ of real numbers is   4+3+3
                                                                         =10
       isomorphic to the multiplicative group $(R^+, \times)$ of positive real

       numbers

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b. Check whether the set of integers Z with the binary operation " $*$ " defined as $a * b = a + b + 1$ for $a, b \in Z$ is a group or not.

c. Define kernel, range and null space of a linear transformation.

5. a. Define Volterra Integral Equation of the first kind.
   b. Reduce the boundary value problem to Fredholm Integral Equation $y''(x) + x\, y(x) = 1$, where $y(0) = 0$, $y(1) = 0$.
   c. Express the above Fredholm Integral Equation into its 2nd kind

<div align="right">2+6+2
=10</div>

6. a. Explain the characteristics of an abelian group?
   b. Let $G = \{1, -1, i, -i\}$, which forms a group under multiplication and Z is the group of all integers under addition. Prove that the mapping f onto G such that $f(x) = i^n$ $\forall n \in Z$ is a homomorphism.

<div align="right">5+5=10</div>

7. a. Express the polynomial $2\, p_2(x) + 3\, p_1(x)$ into the Laguerre polynomial.
   b. Show that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{\pi x}{2}} \sin x$.

   c. Prove that $L_n(0) = 1$.

<div align="right">4+4+2
=10</div>

8. Work out the symmetry group of a square. How many elements does it have? Construct the multiplication table.

<div align="right">2+2+6
=10</div>

$$= = *** = =$$

P.T.O.

[4]