REV-01
MSM/48/16/19

2022/12

SET B

M.Sc. MATHEMATICS
THIRD SEMESTER
MATHEMATICAL METHODS
MSM – 303

[USE OMR SHEET FOR OBJECTIVE PART]

Duration : 3 hrs. Full Marks : 70

(Objective)

Time: 30 min. Marks: 20

Choose the correct answer from the following: ***1X20=20***

1. If $L^{-1}(\frac{S}{S^2-a^2}) = Cosh\ at$ then,
   a. $S > |a|$
   b. $S < |a|$
   c. $S \geq |a|$
   d. $S \leq |a|$

2. $L^{-1}\left(\frac{1}{S^{n+1}}\right) = \frac{t^n}{\Gamma(n+1)}$ then ,
   a. $n \geq -1$
   b. $n > -1$
   c. n is rational
   d. n is positive rational

3. $L(\frac{e^{bt}}{t}) = ..$
   a. 1
   b. 0
   c. $\frac{1}{(S-b)^2}$
   d. $\log\left|\frac{1}{S-b}\right|$

4. Which can't be the eigen value of the equation,
   $y(x) = \lambda \int_a^b k(x,t)\, y(t)\, dt$
   a. $\lambda = 0$
   b. $\lambda = 1$
   c. $\lambda = 2$
   d. None

5. $L^{-1}(0)$
   a. 0
   b. 1
   c. Undefined
   d. None

6. If $F(S)$ is the Fourier transformation of $F(x)$ then Fourier transformation of $F(ax)$ is

a. $\frac{1}{a}F\left(\frac{S}{a}\right)$

b. $F\left(\frac{S}{a}\right)$

c. $F(sa)$

d. $F(sx)$

7. Which of the following can't be a kernel of cosine transformation

a. $Sin\, sx$

b. e^{-isx}

c. e^{sx}

d. All of the above

8. Fourier transform is defined on,

a. $(-\infty,\infty)$

b. $(-\infty,0)$

c. $(0,\infty)$

d. $[0,\infty)$

9. Inverse Laplace transform of $\frac{1}{\sqrt{S}}$ is,

a. $\frac{t^{1/2-1}}{\Gamma(1/2)}$

b. $\frac{t^{1/2}}{\Gamma(1/2)}$

c. Both a. and b.

d. None

10. If $L[f(t)]=\bar{f}(s)$ then $L[f(at)]=$

a. $\frac{1}{a}\bar{f}(s)$

b. $L[f(t)]=\frac{1}{a}\bar{f}(\frac{s}{a})$

c. Both a. and b.

d. None

11. The values of $L\left\{\frac{S\text{int}}{t}\right\}$ is -----

a. $-\tan^{-1}s$

b. $\tan^{-1}s$

c. 0

d. None

12. The value of $L\{3\}$is

a. 0

b. 3

c. $\frac{1}{S}$

d. None

13. Boundary value problems in the theory of ordinary differential equation can lead to integral equations of the type

a. Volterra

b. Fredholm

c. Mellin

d. Laplace

14. If the upper limit of the integral equation is not a constant then the equation is of the type

a. Volterra
b. Fredholm
c. Hankel
d. Holbert

15. $L\left(e^{at}\cos bt\right)$ is

a. $\dfrac{b}{(S-a)^2+b^2}$
b. $\dfrac{1}{(S-a)^2+b^2}$
c. $\dfrac{S}{(S-a)^2+b^2}$
d. $\dfrac{b}{(S+a)^2+b^2}$

16. Linear integral equation of the form ,

$y(x)=f(x)+\lambda\int_a^b k(x,t)\,y(t)\,dt$ is known as Fredholm integral equation of

a. 1st kind
b. 2nd kind
c. 3rd kind
d. None

17. A linear integral equation of the form ,

$y(x)=\lambda\int_a^b k(x,t)\,y(t)\,dt$ is called homogeneous Volterra integral equation of

a. 1st kind
b. 2nd kind
c. 3rd kind
d. All of the above

18. Formula to convert multiple integral

$\int_a^x y(t)\,dt^n$ into a single ordinary integral is,

a. $\int_a^x \dfrac{(x-t)^n}{n!}y(t)\,dt$
b. $\int_a^x \dfrac{(x-t)^{n-1}}{(n-1)!}dt$
c. $\int_a^x \dfrac{(x-t)^n}{n!}dt$
d. None

19. Find $L\left(t^{3/2}\right)$

a. $\dfrac{3\sqrt{\pi}}{4S^{5/2}}$
b. $\dfrac{\sqrt{\pi}}{4S^{5/2}}$
c. $S^{3/2}$
d. None

20. $L(F(t)) = f(S)$ then $L(e^{at}F(t)) = f(S-a)$ is called

a. 1st shifting theorem
b. 2nd shifting theorem
c. Both a. and b.
d. None

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(Descriptive)

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Convert the following differential equation into an integral equation with initial conditions $y(0) = 1,\ \ y\ \ (1) = 1$ **5+5=10**

$$\frac{d^2 y}{dx^2} + \lambda xy = f(x)$$

b. Show that the solution of the Volterra integral equation

$y(x) = 1 + \int_0^x (t - x)y(t)dt$ satisfies the differential equation

$y''(x) + y(x) = 0$

with initial conditions $y(0) = 1,\ \ y'(0) = 1$

2. a. Find $L^{-1}(t^2 e^{-2t} \cos^2 3t)$ **5+5=10**

b. Using Fourier cosine integral representation of the function

$f(x) = e^{-kx}$ show that $\int_0^\infty \frac{\cos wx}{k^2 + w^2} dw = \frac{\pi e^{-kx}}{2k}, x > 0, k > 0$

3. a. Express the function $F(x)$ defined by, **5+5=10**

$$F(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

As a Fourier integral and hence evaluate,

$$\int_0^\infty \frac{Sin\lambda\, Cos\lambda x}{\lambda} d\lambda$$

b. Apply convolution theorem to find,

$$L^{-1}\left\{\frac{1}{(s+4)(s+5)}\right\}$$

4. a. Find Fourier Sine transformation of $f(x)$ if, 5+5=10

$$f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$$

b. Define, *(any two)*
   i. Hankel transformation
   ii. Hilbert transformation
   iii. Mellin transformation

5. a. Solve the integral equation, 5+5=10

$$\phi(x)=x+\int_0^x Sin(x-\xi)\,\phi(\xi)\,d\xi$$

b. Evaluate $L^{-1}\left\{\frac{1}{\sqrt{3s+4}}\right\}$

6. a. Using Laplace transformation solve the following differential equation, 5+5=10

$y''+2y'+5y=e^{-2t}\cos t$ where $y(0)=0, y'(0)=1$

b. Show that, $y(x)=\frac{1}{2}$ is a solution of the integral equation,

$$\int_0^x \frac{y(t)}{\sqrt{x-t}}dt=\sqrt{x}$$

7. a. Convert $y''-Sinx.y'+e^x y=x$ with initial conditions $y(0)=1, y'(0)=1$ to a Volterra integral equation of 2nd kind 5+5=10

b. Find the eigen values of the Fredholm integral equation,

$$y(x)=\lambda\int_0^1 x^2 t\, y(t)\,dt$$

8. **a.** Form an integral equation corresponding to the differential equation, **5+5=10**

$y'' + xy' + y = 0$ with initial conditions, $y(0) = 1, y'(0) = 0$.

b. Find the eigen values and the corresponding eigen function of the integral equation,

$$y(x) = \lambda \int_0^1 (2xt - 4x^2) y(t)\, dt$$

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