REV-01
MSM/02/05

2023/01

SET A

M.Sc. MATHEMATICS
FIRST SEMESTER
LINEAR ALGEBRA
MSM – 105 [REPEAT]
[USE OMR SHEET FOR OBJECTIVE PART]

Duration:3 hrs. **Full Marks:70**

(**Objective**)

Time: 30 min. **Marks:20**

Choose the correct answer from the following: ***1X20=20***

1. The system of equation $5x_1 + 6x_2 = 1$

   $10x_1 + 12x_2 = 3$ gives

   a. unique solution
   b. infinitely many solutions
   c. no solution
   d. Finitely many solutions

2. The system of equation $2x_1 + 3x_2 = 1$

   $4x_1 + 6x_2 = 2$ gives

   a. unique solution
   b. no solution
   c. finitely many solutions
   d. infinitely many solutions

3. If A is a 5×5 real matrix with trace 15 and if 2 and 3 are eigen values of A, each with multiplicity 2, then determinant of A is equal to
   a. 120
   b. 0
   c. 180
   d. 24

4. Let T be a linear transformation on R^2 to itself such that T (1, 0) = (1, 2) and T (1, 1) = (0, 2). Then T(a, b)= is equal to
   a. (a, 2b)
   b. (2a, b)
   c. (a-b, 2a)
   d. (a-b, 2b)

5. Which of the following matrix is positive definite?
   a. $\begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$
   b. $\begin{bmatrix} -3 & 4 \\ 4 & 5 \end{bmatrix}$
   c. $\begin{bmatrix} 8 & -3 \\ -3 & 2 \end{bmatrix}$
   d. $\begin{bmatrix} 2 & 1 \\ 4 & -5 \end{bmatrix}$

6. Which of the following statement is true?
   a. A real inner product space is conjugate symmetric.
   b. A complex inner product space is linear in the second argument.
   c. A complex inner product space is symmetric.
   d. A real inner product space is linear in the first argument.

7. Which of the following set of vectors spans R^2
   a. {(0, 3), (0, -3)}
   b. {(0, 0), (0, 6)}
   c. {(1, 0), (0, 0)}
   d. {(-7,0), (0,-7)}

8. Let W be a subspace of a finite dimensional vector space V then which of the following is true?

a. $Dim\ W \geq DimV$ b. $Dim\ W \leq DimV$

c. $Dim\ W = DimV$ d. none

9. If F is a field of real numbers, the vectors $(a_1, a_2) and\ (b_1, b_2) in\ V_2(F)$ are linearly dependent if

a. $a_1 b_2 - a_2 b_1 = 0$ b. $a_1 b_2 - a_2 b_1 \neq 0$

c. $a_1 b_2 + a_2 b_1 = 0$ d. none

10. Determine the eigen vectors of the matrix A $= \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & c & c \end{bmatrix}$

a. 0, 0 ,0 b. a, b, c

c. a, -b, -c d. none

11. Which of the following statement is true?

a. Zero space is not zero dimensional. b. Roots of characteristic equation are called eigen values.

c. All the straight lines passing through origin are 2 dimensional d. Both (a) and (c)

12. An operator T is said to be non-singular if

a. Ker T $\neq \{0\}$ b. Ker T $= \{0\}$

c. Doesnot depend on ker T d. none

13. Find the characteristic polynomial of the matrix A$=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

a. $t^3 - 18t^2 + 45t$ b. $t^3 + 18t^2 + 45t$

c. $t^3 - 18t^2 - 45t$ d. None

14. Which of the following set of vectors are linearly independent?

a. $\{(2,1,2),(8,4,8)\}$ b. $\{(2,3,5),(4,9,25)\}$

c. {(-1, 2, 1),(3, 0, -1),(-5, 4, 3)} d. none

15. Which of the following transformation is linear if $T: R^2 \to R^2$ defined by

a. $T(x,y) = (x^2, y^2)$ b. $T(x,y) = (x+2, y+3)$

c. $T(x,y) = (x+y, x-y)$ d. $T(x,y) = (0, sinx)$

16. If $f(x,y,z) = x^2 + y^2 + 4z^2 + 2xy + 4xz$, then what is the matrix form?

a. $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 4 \end{bmatrix}$ b. $\begin{bmatrix} 2 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 0 & 4 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 4 \end{bmatrix}$

17. Given ϕ be the linear functional on R^2 defined by $\phi(x,y) = x - 2y$. For any linear operator T on R^2 find $(T'(\phi))(x,y)$ where $T(x,y) = (y,\ x+y)$

a. $-2x - y$ b. $-2x + y$

c. $2x - y$ d. none

18. If T is a linear operator on R^2 defined by $T(x,y) = (x-y,y)$ then $T^2(x,y) = ?$

a. $(x+2y,y)$ b. $(x-2y,y)$

c. $(x-2y,x+2y)$ d. $(x-2y,y+2x)$

19. The set of vector (2,-2) and (2, 2) with respect to standard inner product are

a. orthonormal b. continuous

c. orthogonal d. none

20. Dimension of polynomial of degree ≤ 4 over the field of real is

a. 4 b. 5

c. 3 d. 2

-- --- -

(Descriptive)

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. If W_1 and W_2 be two subspaces of a finite dimensional vector space V such that $W_1 \cap W_2$={0}.Prove that Then $\dim(W_1 + W_2) = \dim W_1 + \dim W_2$. 7+3=10
Prove that a linear transformation $T: V \to W$ is said to be injective if $T(x) = T(y) \Rightarrow x = y \; for \; all \; x, y \in V$.

2. 5+5=10
a. Find the characteristics equation of the following matrix A=$\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and hence findA^{-1} using Cayley-Hamilton theorem.

b. Define characteristicand minimal polynomial of a block matrixFind the minimal polynomial of A=$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

3. a.$T: R^2 \to R^3$ is a linear transformation defined by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, \; 2x_2)$. Find $[T]_{B_1}^{B_2}$. Where, $B_1 = \{(1,0), (0,1)\}$ 4+6=10
$B_2 = \{(1,1,0), (1,-1,0), (0,0,1)\}$are the basis of R^2 and R^3 respectively.
b. Consider the basis $S = \{\alpha_1, \alpha_2, \alpha_3\} of \; R^3$ where $\alpha_1 = (1,1,1), \alpha_2 = (1,1,0), \alpha_3 = (1,0,0)$. Express (2, -3, 5) in terms of the basis $\alpha_1, \alpha_2, \alpha_3$.
If $T: R^3 \to R^2$ be defined as
$T(\alpha_1) = (1,0), \; T(\alpha_2) = (2,-1), \quad T(\alpha_3) = (4,3)$. Find T(2, -3, 5).

4. a. Define dual space and dual basis of a vector space. Define Kronecker delta in a short way. 2+2+6=10

b. Consider the basis $\{v_1 = \{1,-2, \; 3\}, v_2 = \{1,-1,1\}, v_3 = \{2,-4, \; 7\}\}$. Find the basis $\{\phi_1, \phi_2, \phi_3\}$ that is dual to the basis of R^3.

5. a. Define Jordan canonical form of a matrix. If the eigen values $\lambda = 3, 3, 4 \; and \; 5$ find the Jordan Canonical form of the matrix. 3+7=10

b Find all eigen values and eigen vectors of the following matrix A=

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Is A diagonalizable? If yes find P such that $P^{-1}AP$ is diagonal.

6. a. If T, T_1, T_2 are the adjoint operators then prove that $(i)(T_1 + T_2)^* = T_1^* + T_2^*$ (ii) $(T_1T_2)^* = T_2^*T_1^*$ (iii)$(KT_1)^* = \bar{K}T_1^*$ **6+4=10**

b. Prove that two vectors x *and* y in a real inner product space are orthogonal if and only if $\|x + y\|^2 = \|x\|^2 + \|y\|^2$

7. a. Define self adjoint operator.Given T is a self adjoint operator on V. Suppose u *and* v are the eigen vectors of T belonging to distinct eigen values. Then prove that u*and* v are orthogonal. **5+5=10**

b. Using Gram-Schmidt process to the vectors $\alpha_1 = (2\ 0, 1), \alpha_2 = (3, -1, 5), \alpha_3 = (0, 4, 2)$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

8. a. Define bilinear form with an example. Also define alternating and skew symmetric form. **2+2+3+3=10**

b. (i)Find the adjoint$G^*$of G:$C^3 \rightarrow C^3$ *defined by*

$$G(x, y, z) = [2x + (1 - i)y, (3 + 2i)x - 4iz, 2ix + (4 - 3i)y - 3z]$$

(ii) Define normal operator. Is the following matrix normal?

$$A = \begin{bmatrix} 1 & 1 \\ i & 3 + 2i \end{bmatrix}$$

= = *** = =