REV-01
MSM/02/05

2023/01

**SET A**

# M.Sc. MATHEMATICS
## FIRST SEMESTER
## DIFFERENTIAL EQUATION
## MSM – 102 [REPEAT]

[USE OMR SHEET FOR OBJECTIVE PART]

Duration : 3 hrs. Full Marks : 70

## (Objective)

Time: 30 min. Marks: 20

***Choose the correct answer from the following:*** ***1X20=20***

1. $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$ is a linear differential equation of 2nd order,where $P,Q,R$ is a function of
   a. $y$ only
   b. $x$ only
   c. $z$ only
   d. None of the above

2. A function $f(x,y)$ is said to satisfy a Lipschitz condition in a region D in xy-plane if there exists a positive constant $k$ such that
   a. $|f(x,y_2)-f(x,y_1)|\le k|y_2-y_1|$
   b. $|f(x,y_2)-f(x,y_1)|=k|y_2-y_1|$
   c. $|f(x,y_2)-f(x,y_1)|<k|y_2-y_1|$
   d. $|f(x,y_2)-f(x,y_1)|\ge k|y_2-y_1|$

3. An equation of the form $Pdx+Qdy+Rdz=0$ ,where $P,Q,R$ are function of $x,y,z$ is called a
   a. Lagrange equation
   b. Simultaneous differential equation
   c. Charpit equation
   d. Total Differential equation

4. $\sum_{n=0}^{\infty}C_n(x-x_0)^n$ is called a power series in
   a. $(x+x_0)$
   b. $x$
   c. $(x-x_0)$
   d. $-x$

5. Condition of Integrability of a total differential equation is
   a. $P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0$
   b. $P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)-Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0$
   c. $P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)-R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0$
   d. None of the above

6. The Pochhammer symbol is denoted and defined by
   a. $(\alpha)_n=\alpha(\alpha+1)............(\alpha+n+1)$
   b. $(\alpha)_n=\alpha(\alpha+1)............(\alpha+n-1)$
   c. $(\alpha)_n=\alpha+(\alpha+1)+............+(\alpha+n-1)$
   d. $(\alpha)_n=\alpha(\alpha-1)............(\alpha-n-1)$

7. A Power series converges for

a. $|x| < R$      b. $|x| > R$

c. $|x| = R$      d. $|x| \leq R$

8. If $y = e^{-x}$ is a part of complementary function then

a. $1 - P - Q = 0$      b. $1 + P + Q = 0$

c. $P + Qx = 0$      d. $1 - P + Q = 0$

9. Existence Theorem is said to be existence because it says that

a. The initial Value Problem does have solution

b. The initial Value Problem does not have solution

c. No solution

d. None of the above

10. A quasi-linear partial differential equation of order one is of the form $Pp + Qq = R$ ,where $P, Q, R$ are function of $x, y, z$ is known as

a. Charpit's equation      b. Lagrange Equation

c. Monge's equation      d. Legendre's equation

11. The Equation $(1 - x^2)y'' - 2xy' + n(n+1) = 0$ , where $n$ is a positive integer,is called

a. Charpit equation      b. Lagrange equation

c. Monge's equation      d. Legendre equation

12. In Removal of First Derivative Method

a. $u = e^{\frac{1}{2}\int P dx}$      b. $u = e^{-\frac{1}{2}\int P dx}$

c. $u = e^{\frac{1}{2}\int Q dx}$      d. $u = e^{-\frac{1}{2}\int Q dx}$

13. Charpit's Auxillary equation can be written as

a. $\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q} = \frac{dF}{0}$

b. $\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dF}{0}$

c. $\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dF}{0}$

d. $\frac{dp}{f_x - pf_z} = \frac{dq}{f_y - qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q} = \frac{dF}{0}$

14. The differential equation $Pdx + Qdy + Rdz = 0$ is exact if

a. $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial z} = -\frac{\partial R}{\partial y}, \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z}$

b. $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}, \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z}$

c. $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}, \frac{\partial R}{\partial x} = -\frac{\partial P}{\partial z}$

d. $\frac{\partial P}{\partial y} = -\frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}, \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z}$

15. In Variation of Parameter method,if $y = A\cos 2x + BSin2x$ be the complete primitive of the given equation,then $A$ and $B$ are

a. function of $x$

b. constant

c. Variable

d. function of $y$

16. Lipschitz constant of the function $f(x,y) = xy^2$ on the rectangle $R : |x| \le 1, |y| \le 1$ is

a. $3$

b. $-2$

c. $-3$

d. $2$

17. The equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ represents a

a. System of curves in space,the direction cosines of the tangent to any member of this system are proportional to $P,Q,R$.

b. System of curves in space,the direction cosines of the tangent to any member of this system at any point $(x,y,z)$ are proportional to $R,P,Q$.

c. System of curves in space,the direction cosines of the tangent to any member of this system at any point $(x,y,z)$ are proportional to $P,Q,R$.

d. The direction cosines of the tangent to any member of this system at any point $(x,y,z)$ are proportional to $P,Q,R$.

18. In series solution $2x^2\frac{d^2y}{dx^2} + 7x(x+1)\frac{dy}{dx} - 3y = 0$, $x = 0$ is

a. Both

b. Singular Point

c. Absolute Regular Point

d. Ordinary Point

19. The nth approximation of Picard's Theorem is

a. $y_n(x) = y_0 + \int_{x_0}^{x} f(x, y_{n-1})dx$

b. $y_n(x) = y_0 - \int_{x_0}^{x} f(x, y_{n-1})dx$

c. $y_n(x) = y_0 + \int_{x_0}^{x} f(x, y_n)dx$

d. $y_n(x) = y_0 + \int_{x_0}^{x} f(x, y_{n+1})dx$

20. Condition for Frobenious Method is

a. $x = 0$ is Regular point

b. $x = 0$ is Singular point

c. $x = 0$ is an ordinary point

d. None of the above

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## Descriptive

**Time : 2 hrs. 30 mins.** **Marks : 50**

***[ Answer question no.1 & any four (4) from the rest ]***

**1.** **a.** What do you mean by Analytic function & Power series? **2+8=10**

**b.** Solve by Frobenious Method $9x(1-x)y'' - 12y' + 4y = 0$

**2.** **a.** What is geometrical interpretation of Simultaneous differential equation. **3+7=10**

**b.** Solve

$$\frac{xdx}{z^2 - 2yz - y^2} = \frac{dy}{y+z} = \frac{dy}{y-z}$$

**3.** State Existence and Uniqueness Theorem.Prove That **3+7=10**

$y_n = y_0 + \sum_{n=1}^{n}(y_n - y_{n-1})$ must be continuous.

**4.** **a.** What do you known by Known Integral Method **1+9=10**

**b.** Solve $(x+1)\frac{d^2y}{dx^2} - 2(x+3)\frac{dy}{dx} + (x+5)y = e^x$

**5.** **a.** Find the condition of integrability of total differential equation **6+4=10**

**b.** Solve $xdx + ydy + zdz = 0$

**6.** What is Legendre polynomial of 2nd kind.Prove that **2+8=10**

$$y = \frac{1.3.5.........(2n-1)}{n!}\left[x^n - \frac{n(n-1)}{2(2n-1)}x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2.4...........(2n-1)(2n-3)}x^{n-4} + .....\right]$$

**7.** Solve by Variation of Parameter Method **10**

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$$

**8.** Solve: **2×5=10**

**a.** $(y^2z/x)p + xzq = y^2$

**b.** $z(z^2 + xy)(px - qy) = x^4$

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