SET A

M.Sc. PHYSICS
THIRD SEMESTER
THEORY OF RELATIVITY-I
MSP – 304E

[USE OMR SHEET FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. One of the basic postulates of special theory of relativity is
   a. Speed of light in vacuum is the upper limit of speed of any object.
   b. Speed of light is constant only in inertial frame.
   c. Newton's Laws of motion is invariant.
   d. All of these

2. Two photons approach each other with speed c. Their relative velocity will be
   a. c
   b. 0
   c. 2c
   d. c/2

3. A one meter rod is moving along x-axis with a relativistic speed 0.8c. Its moving length will be (in meter)
   a. 30%
   b. 60%
   c. 70%
   d. 90%

4. Consider two frames S and S′ ,where the later one is moving with a relativistic velocity v along a particular direction. Statements: (i) two events are simultaneous in S frame. Then (i) these two events are not simultaneous in S′ frame.
   a. Both (i) & (ii) are true
   b. (i) is false and (ii) is true
   c. (i) is true and (ii) is false
   d. Both (i) & (ii) are false

5. The relativistic kinetic energy of a particle of rest mass M moving with velocity v will be
   a. $\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right] Mc^2$
   b. $\left[1 - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right] Mc^2$
   c. $\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} + 1\right] Mc^2$
   d. $\left[-1 - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right] Mc^2$

6. The Lorentz transformation equation in which the time t of an event in two inertial frame of reference, S & S′, will be
   a. $t' = \frac{-t+\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$
   b. $t' = \frac{t-\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$
   c. $t' = \frac{t+\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$
   d. $t' = \frac{t-\frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}}$

7. Einstein's mass energy relation (E = m c^2) shows that

a. Mass disappear to reappears as energy
b. Mass and energy are two different forms of same entity
c. Energy disappears to reappears as mass
d. All of these

8. A cube moving along one face with high speed will look like a

a. Rectangle
b. rectangular parallelepiped
c. Sphere
d. Cube

9. The relativistic velocity of a particle of mass m and rest m_0 will be

a. $v = c\sqrt{1-\left(\frac{m_0}{m}\right)^2}$
b. $v = c/\sqrt{1-\left(\frac{m_0}{m}\right)^2}$
c. $v = c\sqrt{1+\left(\frac{m_0}{m}\right)^2}$
d. $v = c/\sqrt{1+\left(\frac{m_0}{m}\right)^2}$

10. A young fat girl dances with high velocity, she will appear to her stationary friends

a. Less Fat
b. More Fat
c. Same dimensions
d. some time less and some time more fat

11. The metric component $g_{\theta\theta}$ in the line-element $ds^2 = dr^2 + r^2(d\theta^2 + sin^2\theta\, d\varphi^2)$ is

a. r^2
b. $\alpha^2\, r^2$
c. $r^2\, sin^2\theta$
d. 1

12. The relativistic energy relation in terms of non-relativistic limit becomes

a. $mc^2 = m_0c^2 + \frac{1}{2}m_0v^2$
b. $mc^2 = m_0c^2 + m_0v^2$
c. $m_0c^2 = mc^2 + \frac{1}{2}m_0v^2$
d. $\frac{1}{2}m_0v^2 = mc^2 + m_0c^2$

13. If $V^\mu = (1, 1)$ and the metric tensor $g_{\mu\nu} = \begin{pmatrix}1 & 0\\ 0 & r^2\end{pmatrix}$, then V_μ will be

a. $(1, r^2)$
b. $(r^2, 1)$
c. (1,1)
d. (1,0)

14. A vector V^μ is said to be a null vector if

a. $g_{\mu\nu}V^\mu V^\nu > 0$
b. $g_{\mu\nu}V^\mu V^\nu < 0$
c. $g_{\mu\nu}V^\mu V^\nu = 0$
d. All of these

15. The Kronecker delta δ^μ_ν acts on the vector $A^{\nu\alpha}_\beta$ result

a. $-A^{\mu\alpha}_\beta$
b. $A^{\mu\alpha}_\beta$
c. $A^\mu_{\alpha\beta}$
d. $-A^\mu_{\alpha\beta}$

16. The Christoffel's symbol $\Gamma^\mu_{\mu\nu}$ is

a. $\frac{\partial}{\partial x^\nu}(\ln\sqrt{g})$
b. $\frac{\partial}{\partial x^\nu}(\ln g)$
c. $\frac{\partial}{\partial x^\mu}(-\ln\sqrt{g})$
d. $\frac{\partial}{\partial x^\nu}(-\ln g)$

17. Covariant derivative of a vector results a vector of rank

a. 1
b. 2
c. 0
d. 3

18. The rank of the mixed tensor $\delta^{\tau}_{\alpha}\delta^{\mu}_{\sigma}B^{\sigma}_{\rho\tau}$ is

a. 2
b. 3
c. 1
d. 4

19. The number of non-zero components of Riemann tensor in three dimensions are

a. 6
b. 10
c. 20
d. 15

20. The determinant of the metric tensor $ds^2 = dr^2 + r^2 d\varphi^2$ will be

a. r^2
b. $-r^2$
c. 1
d. -1

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Descriptive

Time : 2 hrs. 30mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Derive an expression of the Riemann curvature tensor $R^{\lambda}_{\mu\nu\sigma}$ in terms of the Christoffel symbols. 6+4=10

   b. Show that the Riemann tensor $R_{\lambda\mu\nu\sigma}$ satisfies the relation: $R_{\lambda\mu\nu\sigma} + R_{\lambda\nu\sigma\mu} + R_{\lambda\sigma\mu\nu} = 0$.

2. a. Find the momentum transformation relations using Lorentz transformation. 5+2+3 =10

   b. Derive the electromagnetic field $F_{\mu\nu}$ using Maxwell's equations.

   c. From the conservation law of electric charge show that surface charge density as measured by a moving observer relative to an inertial frame increases.

3. a. Derive the expression of covariant derivative of a covariant vector $A_{\mu;\nu}$. 4+3+3 =10

   b. Show that the covariant derivative of the fundamental tensor $g_{\mu\nu}$ vanishes.

   c. Prove that $\Gamma^{\mu}_{\mu\nu} = \frac{\partial}{\partial x^{\nu}}\left(\log\sqrt{g}\right)$.

4. a. Find the Christoffel symbol associated with the metric tensor $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$. 6+2+2 =10

   b. What will be the determinant of the above metric tensor?

   c. Find the inverse metric tensor for the above line-element.

5. a. Using Lorentz transformation of electromagnetic field tensor show that $c^2B^2 - E^2$ is invariant. 4+3+3 =10

   b. Find an expression of moving length of a rod w. r. t. an inertial frame.

   c. A 5m long rod lying stationary in a frame. What will be its length as measured by a moving observer with a relativistic velocity 0.8c?

6. a. Show that addition of two contravariant tensor of same rank is also a tensor of the same rank. 4+3+3 =10

b. If $\bar{A}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\nu}} A^{\nu}$ then prove that $A^{\mu} = \frac{\partial x^{\mu}}{\partial \bar{x}^{\nu}} \bar{A}^{\nu}$.

c. Show that the mixed tensor δ^{μ}_{ν} is an invariant.

7. a. Show that a second rank contravariant tensor is expressible as a sum of symmetric and anti-symmetric tensor. 3+2+3

b. Find the metric tensor $ds^2 = dx^2 + dy^2 + dz^2$ in spherical coordinates.

c. Find the conjugate metric tensor of the above metric tensor.

8. a. Derive the mass-energy equivalence relation E=$m\ c^2$ 6+2+2 =10

b. Prove the momentum-energy relation $E^2 = P^2c^2 + m^2c^4$.

c. A particle of rest mass 2kg is moving with a relativistic velocity 0.8c. Find its momentum.

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