2023/01

REV-01
MSP/41/21/26

SET B

M.Sc. PHYSICS
FIRST SEMESTER
ELECTRODYNAMICS
MSP – 104
[USE OMR SHEET FOR OBJECTIVE PART]

Full Marks: 35

Duration: 1:30 hrs.

(**Objective**)

Marks: 10

Time: 15 mins.

1X10=10

Choose the correct answer from the following:

1. The allowed gauge transformations for the scalar and vector potentials are
   a. $A' = A + \nabla\lambda, V' = V - \frac{\partial\lambda}{\partial t}$
   b. $A' = A - \nabla\lambda, V' = V + \frac{\partial\lambda}{\partial t}$
   c. $A' = A + \nabla\lambda, V' = V + \frac{\partial\lambda}{\partial t}$
   d. $A' = A - \nabla\lambda, V' = V - \frac{\partial\lambda}{\partial t}$

2. In the Coulomb gauge, we pick
   a. $\nabla \cdot A = 0$
   b. $\nabla \cdot A = -\mu_0\epsilon_0 \frac{\partial V}{\partial t}$
   c. $\nabla \cdot A = \mu_0\epsilon_0 \frac{\partial V}{\partial t}$
   d. None of these

3. The invariant interval is
   a. $c^2t^2 + x^2 + y^2 + z^2$
   b. $c^2t^2 - x^2 - y^2 - z^2$
   c. $-c^2t^2 - x^2 - y^2 - z^2$
   d. $-c^2t^2 + x^2 + y^2 + z^2$

4. Four-vector potential is
   a. $\left(\frac{V}{c}, A_x, A_y, A_z\right)$
   b. (V, A_x, A_y, A_z)
   c. $\left(V, \frac{A_x}{c}, \frac{A_y}{c}, \frac{A_z}{c}\right)$
   d. $\left(\frac{V}{c}, \frac{A_x}{c}, \frac{A_y}{c}, \frac{A_z}{c}\right)$

5. The potential for a dipole goes like (at large distances)
   a. $V \sim \frac{1}{r}$
   b. $V \sim \frac{1}{r^2}$
   c. $V \sim \frac{1}{r^3}$
   d. $V \sim \frac{1}{r^4}$

6. In 2D, the Laplace's equation reads as (symbols have their usual meanings)
   a. $\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} = 0$
   b. $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$
   c. $\frac{\partial^2 V}{\partial x \partial y} + \frac{\partial^2 V}{\partial y \partial x} = 0$
   d. $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \rho/\epsilon_0$

7. If V has azimuthal symmetry, it will be independent of
   a. r
   b. θ
   c. ϕ
   d. both r and θ

8. $f(z,t)$ represents a wave of fixed shape traveling in the z direction at speed v. One of the following forms does not represent a wave. Which one?

a. $Ae^{-b(z-vt)^2}$ b. $Ae^{b(bz^2+vt)}$

c. $\frac{A}{b(z-vt)^2}$ d. $A\sin[b(z-vt)]$

9. For an electromagnetic (EM) plane wave $\hat{n}$ and $\hat{k}$ are the polarization vector and the direction of wave vector, respectively. One of the followings is true. Choose the right one.

a. $\hat{n}\times\hat{k}=0$ b. $\hat{n}\cdot\hat{k}=0$

c. $\hat{n}\cdot\hat{k}\neq 0$ d. None of these

10. If R is the reflection coefficient and T is the transmission coefficient, then

a. $R+T=1$ b. $R+T<1$

c. $R+T>1$ d. $R+T=0$

Descriptive

Time : 1 hr.15 mins. Marks: 25

[Answer question no.1 & any two (2) from the rest]

1. The intensity of sunlight hitting the earth is about 1300 W/m^2. If sunlight strikes a perfect absorber, what pressure does it exert? How about a perfect reflector? What fraction of atmospheric pressure does this amount to? 2+2+1 =5

2. A sphere of radius R, centered at the origin, carries charge density 2+2+6 =10

$$\rho(r,\theta) = k\frac{R}{r^2}(R-2r)\sin\theta,$$

Where k is a constant, and r, θ are the usual spherical coordinates.

i. Show that for the approximate potentials for the points far from the sphere, the monopole and dipole contributions are zero. 10

ii. Find the quadrupole contribution to the approximate potential for points on the z axis, far from the sphere.

[Help: $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\int (r')^n P_n(\cos\theta')\rho(\vec{r'})\,d\tau'$]

3. i. The potential $V_0(\theta) = k\sin^2\left(\frac{\theta}{2}\right)$ is specified on the surface of a hollow sphere, of radius R. Find the potential *inside* the sphere. 5+5=10

ii. The potential $V_0(\theta) = k\sin^2\left(\frac{\theta}{2}\right)$ is again specified on the surface of a hollow sphere, of radius R. Find the potential *outside* the sphere. 10

[Hint: $V(r,\theta) = \sum_{l=0}^{n}\left(A_l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$, think about the cases $r \to 0$ and $r \to \infty$, which factor is going to blow up. Try to write down the potential in terms of the Legendre polynomials and then equate. First few Legendre polynomials: $P_0(x) = 1, P_1(x) = x, P_2(x) = (3x^2-1)/2$]

4. The potentials for a particular charge and current distributions are 5+4+1 =10

$$V = 0, \qquad A = \begin{cases} \frac{\mu_0 k}{4c}(ct-|x|)^2\hat{z} & \text{for} |x| < ct \\ 0 & \text{for} |x| > ct \end{cases}$$

Where k is a constant and $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$.

(i) Find the electric and magnetic fields and plot them as a function of x. 10
$\left[E = \nabla V - \frac{\partial A}{\partial t}\right]$

(ii) Show that the computed electric and magnetic fieldssatisfy the Maxwell's first two equations.

(iii) What is the charge density in the present case?

5. a. Event A happens at point $(x_A = 5, y_A = 3, z_A = 0)$ and at time t_A given by $ct_A = 15$; event B occurs at (10,8,0) and $ct_B = 5$, both in system S. **2+2+3+ 2+1=10**
(i) What is the invariant interval between A and B?
(ii) Is there an inertial system in which they occur simultaneously? If so, find its velocity (magnitude and direction) relative to S.
(iii) Is there an inertial system in which they occur at the same point? If so, find its velocity relative to S.

b. (i) Write down matrix form of the field tensor $F^{\mu\nu}$. **10**
(ii) Write down the Maxwell's equations in terms of the field tensor $F^{\mu\nu}$.

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