B.Sc. CHEMISTRY
THIRD SEMESTER
VECTOR ANALYSIS
BSM – 732 [REPEAT]
[USE OMR FOR OBJECTIVE PART]

Duration: 1.30 hrs. **Full Marks: 35**

[PART-A: Objective]

Time: 15 min. **Marks: 10**

Choose the correct answer from the following: ***1X10=10***

1. The value of $-\hat{i}\,(\hat{k} \times \hat{j})$ is
   a. -1 b. 1
   c. 0 d. None of these

2. If two vectors $\vec{a}$ and $\vec{b}$ are parallel then $\vec{a} \times \vec{b}$ is
   a. 0 b. A non-zero vector
   c. any vector d. None of these

3. The unit tangent vector at any point of the curve $x = 3\cos\theta, y = 3\sin\theta, z = 4\theta$ is:
   a. 3 b. 4
   c. 5 d. 6

4. If $\theta = 3x^2y - y^3z^2$, then gradient at the point $(1,-2,-1)$ is
   a. $-12\hat{i} + 9\hat{j} + 16\,\hat{k}$ b. $-12\hat{i} - 9\hat{j} + 16\,\hat{k}$
   c. $-12\hat{i} - 9\hat{j} - 16\,\hat{k}$ d. None of these

5. If $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)j$, the curl$(\vec{F})$ is
   a. 0 b. $4y\,\hat{k}$
   c. $\hat{i} + \hat{j} + \hat{k}$ d. None of these

6. The unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ is
   a. $\sqrt{14}(2\hat{i} + 3\hat{j} + \hat{k})$ b. $-2\hat{i} - 3\hat{j} + \hat{k}$
   c. $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$ d. None of these

7. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect?
   a. $\vec{b} = \lambda\vec{a}$, for some λ b. $\vec{b} = \pm\vec{a}$
   c. Both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes. d. None of these

8. The angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is

a. $\cos^{-1}\left(\frac{1}{3}\right)$ b. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

c. $\cos^{-1}(\sqrt{3})$ d. None of these

9. If $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$ then $\frac{d\vec{r}}{dt}$ at $t = 2$ is

a. $12\hat{i} + 2\hat{j} - 20\hat{k}$ b. $12\hat{i} + 2\hat{j} + 20\hat{k}$

c. $12\hat{i} - 2\hat{j} + 20\hat{k}$ d. None of these

10. If ∇ denotes vector differential operator and ϕ is any scalar function then which of the following is true?

a. $\nabla\phi$ = grad ϕ b. $\nabla\phi \neq$ grad ϕ

c. $\nabla\phi$ and grad ϕ are unrelated d. None of these

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PART-B : Descriptive

Time : 1 hr. 15 min. Marks : 25

[Answer question no.1 & any two (2) from the rest]

1. Find the vector triple product for the vectors 5

$\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$

2. a. Let $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Find: 2×3+4 =10

(i) $\vec{a} \times \vec{b}$

(ii) $|\vec{a} \times \vec{b}|$

(iii) Unit vector perpendicular to $\vec{a}$ and $\vec{b}$.

b. Find the angle between the vector $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$.

3. a. Find the divergence and curl of 6+4=10

$$\vec{V} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$$

at $(2, -1, 1)$.

b. Find the rate of change of $\phi = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2$ at the point $(1, 1, 1)$.

4. a. If $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xz + yz + xy$. Prove that grad u, grad v and grad w are coplanar vector. 6+4=10

b. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ then show that - $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$.

5. a. Find the directional derivatives of $x^2y^2z^2$ at the point $(1, 1, -1)$ in the direction of the curve $x = e^t$, $y = \sin 2t + 1$ and $z = 1 - \cos t$ at $t = 0$ 6+4=10

b. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the positive vectors of the vertices A, B, C respectively of a triangle ΔABC then prove that -

$(\vec{a} - \vec{c}) + (\vec{c} - \vec{b}) + (\vec{b} - \vec{a}) = 0$

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