SET A

B.Sc. PHYSICS
FIRST SEMESTER
INTRODUCTION TO MATHEMATICAL PHYSICS
BSP – 101

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(PART-A: Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$, then $div\,\vec{r}$ is
   a. 2 b. 3
   c. -3 d. -2

2. For the right handed system of three coplanar vectors
   $\vec{A} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{B} = 3\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{C} = -\hat{i} + 4\hat{j} + m\hat{k}$, the value of m must be equal to
   a. 5 b. 8
   c. 0 d. 6.5

3. A vector points **A** vertically upward and point **B** towards north. The vector product **A×B** is
   a. along west b. along east
   c. zero d. vertically downward

4. The unit normal to $x^2 + y^2 + z^2 = 5$ at the point $(0,1,2)$ is
   a. $\frac{1}{\sqrt{5}}(\hat{i} + \hat{j} + \hat{k})$ b. $\frac{1}{\sqrt{5}}(\hat{i} + \hat{j} - \hat{k})$
   c. $\frac{1}{\sqrt{5}}(\hat{i} + 2\hat{k})$ d. $\frac{1}{\sqrt{5}}(\hat{i} - \hat{j} + \hat{k})$

5. Gauss's theorem is the relationship between
   a. Surface and volume integral b. line and surface integral
   c. line and volume integral d. none of these

6. If $\phi = yz$, then its gradient is
   a. $z\hat{j} + y\hat{k}$ b. 0
   c. $y\hat{j} + z\hat{k}$ d. $\hat{i} + \hat{j} + \hat{k}$

7. The electric field due to a point charge Q is expressed $\vec{E} = \frac{Q\hat{r}}{4\pi\varepsilon_0 r^2}$, then the divergence of electric field due to that point charge is

a. $\frac{3Q}{4\pi\varepsilon_0 r^2}$

b. $\frac{2Q}{4\pi\varepsilon_0 r}$

c. 0

d. $\frac{3Q}{4\pi\varepsilon_0 r}$

8. The direction of $grad\phi$ is

a. Tangential to level surfaces

b. Normal to level surface

c. Inclined at 45^0 to level surface

d. Arbitrary

9. If $\vec{A} = x\hat{i}$ and $\vec{B} = y\hat{j}$ then $\nabla(\vec{A}.\vec{B})$ is equal to

a. $x\hat{i} + y\hat{j}$

b. 0

c. $\frac{1}{2}yx^2\hat{i} + \frac{1}{2}xy^2\hat{j}$

d. 2

10. The flux leaving any closed surface per unit volume in a vector field $\vec{A}$ is called

a. $grad\ \vec{A}$

b. $div\ \vec{A}$

c. $curl\ \vec{A}$

d. $flux\ \vec{A}$

11. Which of the following vectors are perpendicular to each other?

(i) $2\hat{i} - 2j + 4\hat{k}$, (ii) $10\hat{i} + 8\hat{j} + 12\hat{k}$ and

(iii) $3\hat{i} + 11\hat{j} + 4\hat{k}$

a. (i) And (ii)

b. (ii) And (iii)

c. (iii) And (i)

d. None of these

12. If for two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then angle between $\vec{a}$ and $\vec{b}$ is

a. 0

b. $\frac{\pi}{2}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{3}$

13. If $\vec{F} = grad(2x^2 - 3y^2 + 4z^2)$, then $curl\,\vec{F}$ is

a. $4x - 6y + 8z$

b. $4x\hat{i} - 6yj + 8z\hat{k}$

c. 0

d. 3

14. Order of differential equation whose solution $y = ae^x + be^{2x} + ce^{3x}$ will be

a. 1

b. 2

c. 3

d. 0

15.

I. $\frac{1}{f(D)}x^m$ will be equal to

a. $[F(D)]^{-1}x^m$

b. $F(D)x^m$

c. $mF(D)x^{m-1}$

d. $mx^{m-1}[F(D)]^{-1}$

16. What is the wronskian determinant of x^2, x^3

a. $2x^4$

b. x^4

c. $3\,x^4$

d. $4\,x^4$

17. The value of α so $e^{\alpha y^2}$ that is an I.F. of the equation $(e^{\frac{-y^2}{2}} - xy)dy - dx = 0$

a. -1

b. 1

c. $\frac{1}{2}$

d. $-\frac{1}{2}$

18. General solution of linear differential equation of first order $\frac{dx}{dy} + Px = Q$

a. $ye^{\int P.dx} = \int Qe^{\int P.dx}dx$

b. $xe^{\int P.dy} = \int Qe^{\int P.dy}dy + C$

c. $y = \int Qe^{\int P.dx}dx + C$

d. $x = \int Qe^{\int P.dy}dy + C$

19. Particular integral of $y'' + 2y' - 3y = e^{2x}$ is

a. $-\frac{1}{5}e^{2x}$

b. $\frac{1}{5}e^{2x}$

c. $-\frac{1}{5}$

d. $-\frac{1}{5}$

20. When y = f(x) + c g(x) is the solution of an ordinary differential equation then

a. f is called the particular integral (P.I.) and g is called the complementary function (C.F.)

b. f is called the complementary function (C.F.) and g is called the particular integral (P.I.).

c. f is called the complementary function (C.F.) and particular function (P.I.)

d. g is called the complementary function (C.F.) and particular function (P.I.)

5. Define Laplacian operator in curvilinear co-ordinate system. (i) In curvilinear co-ordinate show that the differential of an arc length is $(ds)^2 = h_1^2(du)^2 + h_2^2(dv)^2 + h_3^2(dw)^2$ (ii) If u, v, w are orthogonal curvilinear co-ordinates, show that $\frac{\partial \bar{r}}{\partial u}, \frac{\partial \bar{r}}{\partial v}, \frac{\partial \bar{r}}{\partial w}$ and $\nabla u, \nabla v, \nabla w$ are reciprocal system of vectors. **2+4+4 =10**

6. State Stoke's theorem. Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and c is its boundary. **2+8=10**

7. **2+4+4 =10**
   i. If $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate the $\oint \vec{A}.dr$ from (0, 0, 0) to (1, 1, 1) along the curve C.
   ii. Evaluate $\iint_s (yz\hat{i} + zx\hat{j} + xy\hat{k}).\vec{ds}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.
   iii. If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, the evaluate $\iiint_V \nabla \times \vec{F}\, dV$, where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.

8. **7+3=10**
   i. Establish the relation $curl curl \vec{f} = \nabla div \vec{f} - \nabla^2 \vec{f}$
   ii. Prove that for every vector field $\vec{V}$, $div(curl \vec{V}) = 0$.

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Descriptive

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. 3+3+4 =10

Solve (i) $(1+e^{\frac{x}{y}})+e^{\frac{x}{y}}(1-\frac{x}{y})\frac{dy}{dx}=0$

(i) Solve the differential equation $\frac{d^2x}{dt^2}+\frac{g}{l}x=\frac{g}{l}L$ where g, l, L are constants subject to the conditions

$$x=a, \frac{dx}{dt}=0 \text{ at } t=0.$$

2. 5+5=10

i. Write the characteristic of scalar triple product.

ii. Prove that [**a+b, b+c, c+a**]=2[**a,b,c**].

iii. Prove that the diagonal of a parallelogram bisect each other.

3. Solve (i) $(x+2y)(dx-dy)=dx+dy$ 3+4+3 =10

(ii) $\frac{dy}{dx}=\frac{y+1}{(y+2)e^y-x}$

(ii) Find the value of λ, for the differential equation $(xy^2+\lambda x^2 y)dx+(x+y)x^2dy=0$ is exact.

4. i. Prove that the altitudes of a triangle are concurrent. 4+4+2 =10

ii. Find the value of n for which the vector $r^n \vec{r}$ is solenoidal, where $r = x\hat{i}+y\hat{j}+z\hat{k}$.

iii. Define curl of a vector function.