SET A

B.Sc. PHYSICS
FIRST SEMESTER
MATHEMATICAL PHYSICS-I
BSP – 101 [REPEAT]
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. Full Marks: 70

(**PART-A: Objective**)

Time: 20 min. Marks: 20

Choose the correct answer from the following: ***1×20=20***

1. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$, then $div\,\vec{r}$ is
   a. 2 b. 3
   c. -3 d. -2

2. For the right handed system of three coplanar vectors $\vec{A} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{B} = 3\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{C} = -\hat{i} + 4\hat{j} + m\hat{k}$, the value of m must be equal to
   a. 5 b. 8
   c. 0 d. 6.5

3. A vector points **A** vertically upward and point **B** towards north. The vector product **A×B** is
   a. along west b. along east
   c. zero d. vertically downward

4. The unit normal to $x^2 + y^2 + z^2 = 5$ at the point $(0,1,2)$ is
   a. $\frac{1}{\sqrt{5}}(\hat{i} + \hat{j} + \hat{k})$ b. $\frac{1}{\sqrt{5}}(\hat{i} + \hat{j} - \hat{k})$
   c. $\frac{1}{\sqrt{5}}(\hat{i} + 2\hat{k})$ d. $\frac{1}{\sqrt{5}}(\hat{i} - \hat{j} + \hat{k})$

5. Gauss's theorem is the relationship between
   a. Surface and volume integral b. line and surface integral
   c. line and volume integral d. none of these

6. If $\phi = yz$, then its gradient is
   a. $z\hat{j} + y\hat{k}$ b. 0
   c. $y\hat{j} + z\hat{k}$ d. $\hat{i} + \hat{j} + \hat{k}$

7. The electric field due to a point charge Q is expressed $\vec{E} = \frac{Q\hat{r}}{4\pi\varepsilon_0 r^2}$, then the divergence of electric field due to that point charge is

a. $\frac{3Q}{4\pi\varepsilon_0 r^2}$

b. $\frac{2Q}{4\pi\varepsilon_0 r}$

c. 0

d. $\frac{3Q}{4\pi\varepsilon_0 r}$

8. The direction of $grad\phi$ is

a. Tangential to level surfaces

b. Normal to level surface

c. Inclined at 45^0 to level surface

d. Arbitrary

9. If $\vec{A} = x\hat{i}$ and $\vec{B} = y\hat{j}$ then $\nabla(\vec{A}.\vec{B})$ is equal to

a. $x\hat{i} + y\hat{j}$

b. 0

c. $\frac{1}{2}yx^2\hat{i} + \frac{1}{2}xy^2\hat{j}$

d. 2

10. The flux leaving any closed surface per unit volume in a vector field $\vec{A}$ is called

a. $grad\ \vec{A}$

b. $div\ \vec{A}$

c. $curl\ \vec{A}$

d. $flux\ \vec{A}$

11. Which of the following vectors are perpendicular to each other?

(i) $2\hat{i} - 2j + 4\hat{k}$, (ii) $10\hat{i} + 8\hat{j} + 12\hat{k}$ and (iii) $3\hat{i} + 11\hat{j} + 4\hat{k}$

a. (i) And (ii)

b. (ii) And (iii)

c. (iii) And (i)

d. None of these

12. If for two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then angle between $\vec{a}$ and $\vec{b}$ is

a. 0

b. $\frac{\pi}{2}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{3}$

13. If $\vec{F} = grad(2x^2 - 3y^2 + 4z^2)$, then $curl\ \vec{F}$ is

a. $4x - 6y + 8z$

b. $4x\hat{i} - 6yj + 8z\hat{k}$

c. 0

d. 3

14. Order of differential equation whose solution $y = ae^{x} + be^{2x} + ce^{3x}$ will be

a. 1
b. 2
c. 3
d. 0

15.

I. $\frac{1}{f(D)} x^{m}$ will be equal to

a. $[F(D)]^{-1} x^{m}$
b. $F(D)x^{m}$
c. $mF(D)x^{m-1}$
d. $mx^{m-1}[F(D)]^{-1}$

16. What is the wronskian determinant of x^2, x^3

a. $2x^4$
b. x^4
c. $3x^4$
d. $4x^4$

17. The value of α so $e^{\alpha y^2}$ that is an I.F. of the equation $(e^{\frac{-y^2}{2}} - xy)dy - dx = 0$

a. -1
b. 1
c. $\frac{1}{2}$
d. $-\frac{1}{2}$

18. General solution of linear differential equation of first order $\frac{dx}{dy} + Px = Q$

a. $ye^{\int P.dx} = \int Qe^{\int P.dx} dx$
b. $xe^{\int P.dy} = \int Qe^{\int P.dy} dy + C$
c. $y = \int Qe^{\int P.dx} dx + C$
d. $x = \int Qe^{\int P.dy} dy + C$

19. Particular integral of $y'' + 2y' - 3y = e^{2x}$ is

a. $-\frac{1}{5}e^{2x}$
b. $\frac{1}{5}e^{2x}$
c. $-\frac{1}{5}$
d. $-\frac{1}{5}$

20. When y = f(x) + c g(x) is the solution of an ordinary differential equation then

a. f is called the particular integral (P.I.) and g is called the complementary function (C.F.)
b. f is called the complementary function (C.F.) and g is called the particular integral (P.I.).
c. f is called the complementary function (C.F.) and particular function (P.I.)
d. g is called the complementary function (C.F.) and particular function (P.I.)

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Descriptive

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. (a) Define Wronskian. 2+2+4+2=10

If $y_1 = e^{-x}\cos x,\ y_2 = e^{-x}\sin x$

(b) Find Wronskian determinant.

(c) Verify that the solutions satisfy the differential equation $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y = 0$.

(d) Show by Wronskian test the solutions are independent.

2. Solve (i) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$ 5+5=10

(ii) Solve the differential equation $\frac{d^2x}{dt^2} + \frac{g}{l}x = \frac{g}{l}L$ where g, l, L are constants subject to the conditions

$$x = a, \frac{dx}{dt} = 0 \text{ at } t = 0.$$

3. Solve (i) $(x+2y)(dx - dy) = dx + dy$ 3+4+3=10

(ii) $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$

(iii) Find the value of λ, for the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact.

4. i. Prove that the altitudes of a triangle are concurrent. 4+4+2=10

ii. Find the value of n for which the vector $r^n \vec{r}$ is solenoidal, where $r = x\hat{i} + y\hat{j} + z\hat{k}$.

iii. Define curl of a vector function.

5. Define Laplacian operator in curvilinear co-ordinate system. Deduce an expression for gradient of a continuously differentiable vector point function in a curvilinear coordinates. **2+8=10**

6. State Stoke's theorem. Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and c is its boundary. **2+8=10**

7. (i) If $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate the $\oint \vec{A}.dr$ from (0, 0, 0) to (1, 1, 1) along the curve C. **2+4+4 =10**

(ii) Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}).\vec{ds}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.

(iii) If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, the evaluate $\iiint_V \nabla \times \vec{F}\, dV$, where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.

8. (i) Establish the relation $curl curl \vec{f} = \nabla div \vec{f} - \nabla^2 \vec{f}$ **7+3=10**

(ii) Prove that [a+b, b+c, c+a]=2[a,b,c].

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