REV-01
BSP/03/08

2023/01

**SET A**

# B.Sc. PHYSICS
## FIRST SEMESTER
## INTRODUCTION TO MATHEMATICAL PHYSICS
## BSP – 101 IDMJ
[USE OMR FOR OBJECTIVE PART]

**Duration: 3 hrs.** **Full Marks: 70**

( **PART-A: Objective** )

**Time: 30 min.** **Marks: 20**

***Choose the correct answer from the following:*** ***1×20=20***

1. The position vector of a particle is denoted with $\vec{r}$. The velocity $\vec{v}$ would be
   a. $\frac{d\vec{r}}{dt}$
   b. $\frac{d^2\vec{r}}{dt^2}$
   c. $\frac{d^3\vec{r}}{dt^3}$
   d. $\frac{d^4\vec{r}}{dt^4}$

2. The condition for a vector $\vec{A}$ to be solenoidal is
   a. $\nabla \times \vec{A} = 0$
   b. $\nabla \cdot \vec{A} = 0$
   c. $\nabla \vec{A} = 0$
   d. $\nabla^2 \vec{A} = 0$

3. The projection of a vector $\vec{A}$ along the $x$-direction is
   a. $\vec{A} \cdot \hat{x}$
   b. $\vec{A} \cdot \hat{y}$
   c. $\vec{A} \cdot \hat{z}$
   d. $\vec{A} \times \hat{x}$

4. Let $\vec{F}$ be the force on a particle moving along $C$. Then $\int_C \vec{F} \cdot d\vec{r}$ represents
   a. velocity of the particle
   b. projection of $\vec{F}$ in the direction of the position vector of the particle
   c. work done by the force
   d. acceleration of the particle

5. If $\nabla \times (\vec{A} \times \vec{B}) = 0$, then one of the followings is true.
   a. $\vec{A}$ is irrotational but not $\vec{B}$
   b. $\vec{B}$ is irrotational but not $\vec{A}$
   c. $\vec{A} \times \vec{B}$ is irrotational
   d. Both $\vec{A}$ and $\vec{B}$ are irrotational

6. $d(\vec{A} \times \vec{B}) =$
   a. $d\vec{A} \times \vec{B}$
   b. $d\vec{A} \times d\vec{B}$
   c. $\vec{A} \times d\vec{B}$
   d. $d\vec{A} \times \vec{B} + \vec{A} \times d\vec{B}$

7. Choose the correct statement.
   a. $\nabla \cdot \vec{A} = \vec{A} \cdot \nabla$
   b. $\nabla \cdot \vec{A} = -\vec{A} \cdot \nabla$
   c. Both (a) and (b) are true
   d. $\nabla \cdot \vec{A} \neq \vec{A} \cdot \nabla$

8. A unit vector perpendicular to the surface $f(x, y, z) = c$ ($c$ is a constant, independent of $x, y, z$) is
   a. $\nabla f$
   b. $\frac{\nabla f}{|\nabla f|}$
   c. $\nabla f \cdot \nabla f$
   d. $\nabla f |\nabla f|$



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9. A vector field $\vec{A}$ is conservative if and only if $\nabla \times \vec{A} = 0$, or equivalently ($\phi$, a scalar potential)

a. $\vec{A} = \nabla\phi$ b. $\nabla \cdot \vec{A} = 0$

c. $\nabla\vec{A} = \phi$ d. $\phi = \nabla \cdot \vec{A}$

10. The elemental volume in the Cartesian coordinate is

a. $dxdydz$ b. $dx$

c. $dy$ d. $dz$

11. What is the wronskian determinant of $x^2, x^3$

a. $2x^4$ b. $x^4$

c. $3\ x^4$ d. $4\ x^4$

12. The complementary function of the differential equation $(D^2+6D+9)y=5e^{3x}$ is

a. $(C_1+C_2x)e^{-3x}$ b. $(C_1+C_2)e^{-3x}$

c. $(C_1+C_2x)e^{3x}$ d. $(C_1+C_2y)e^{3x}$

13. If m-1 and m+2 are factors of auxiliary equation of $y''+y'-2y=0$ then general solution is

a. $Ae^{-x}+Be^{2x}$ b. $e^{-x}+e^{2x}$

c. $Ae^{x}+Be^{-2x}$ d. $e^{x}+e^{2x}$

14. Two differentiable fuction $Y_1(x)$ and $Y_2(x)$ are said to be linearly dependent if

a. $W(Y_1, Y_2x)=0$ b. $W(Y_1, Y_2x)\neq 0$

c. $W(Y_1, Y_2x)=1$ d. $W(Y_1, Y_2x)\neq 0$

15. $\dfrac{1}{f(D)}x^m$ will be equal to

a. $[F(D)]^{-1}x^m$ b. $F(D)x^m$

c. $mF(D)x^{m-1}$ d. $mx^{m-1}[F(D)]^{-1}$

16. If **A** and **B** are (3,4,5) and (6, 8,9) , then product of the vectors **AB** is -----------

a. 3i+ 4j+ 9k b. 3i-4j+9k

c. -3i+4j+9k d. 3i+4j-9k

17. For the right handed system of three coplanar vectors $\vec{A}=\hat{i}-\hat{j}-2\hat{k}$, $\vec{B}=3\hat{i}+5\hat{j}+6\hat{k}$, $\vec{C}=-\hat{i}+4\hat{j}+m\hat{k}$, the value of m must be equal to

a. 5 b. 8

c. 0 d. 6.5

18. A vector points **A** vertically upward and point **B** towards north. The vector product **A×B** is

a. along west b. along east

c. zero d. vertically downward

19. If for two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then angle between $\vec{a}$ and $\vec{b}$ is

a. 0

b. $\frac{\pi}{2}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{3}$

20. Which of the following is not the Axial vector

a. Torque

b. Angular Velocity

c. Angular Momentum

d. Acceleration

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## (Descriptive)

Time : 2 hrs. 30 min. Marks : 50

*[Answer question no.1 & any four (4) from the rest]*

1. (a) $\dfrac{dy}{dx} = \dfrac{y+1}{(y+2)e^y - x}$ 5+5=10

(b) An equation relating to stability of an areoplane is $\dfrac{dv}{dt} = g\cos\alpha - kv$,

where $v$ is the velocity, $g, \alpha, k$ being constants. Find an expression for the velocity if $v=0$ when $t=0$.

2. (a) Determine the constant $a$ so that the vector $\vec{V} = (x+3y)\hat{\imath} + (y-2z)\hat{\jmath} + (x+az)\hat{k}$ is solenoidal. 5+5=10

(b) Solve the differential equation $\dfrac{d^2x}{dt^2} + \dfrac{g}{l}x = \dfrac{g}{l}L$ where $g, l, L$ are constants subject to the conditions

$$x = a, \frac{dx}{dt} = 0 \text{ at } t = 0.$$

3. a) Evaluate $\nabla^2(\ln r)$. 5+5=10

b) If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, the evaluate $\iiint_V \nabla \times \vec{F}\, dV$, where V is the closed region bounded by the planes $x=0, y=0, z=0$ and $2x+2y+z=4$.

4. If $\vec{A} = (3x^2 + 6y)\,\hat{\imath} = 14yz\hat{\jmath} + 20xz^3\hat{k}$, evaluate $\int_C \vec{A}\cdot d\vec{r}$ along the following paths 4+6=10

(a) $x = t, y = t^2, z = t^3$.

(b) the straight lines from (0,0,0) to (1,0,0), then to (1,1,0), and then to (1,1,1).

5. Evaluate $\iint \vec{A}\cdot\hat{n}\,dS$, where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and $S$ is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. **10**

6. Solve: **5+5=10**

i. $\dfrac{d^3y}{dx^3} - 7\dfrac{d^2y}{dx^2} + 10\dfrac{dy}{dx} = e^{2x}\sin x$

ii. $\dfrac{dy}{dx} = \dfrac{y+1}{(y+2)e^y - x}$

7. i. Prove that **[a+b, b+c, c+a]=2[a,b,c]**. **3+4+3 =10**

ii. Prove that $\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$

ii. If $y_1 = e^{-x}\cos x,\ y_2 = e^{-x}\sin x$

Verify that the solutions satisfy the differential equation $\dfrac{d^2y}{dx^2} + \dfrac{2dy}{dx} + 2y = 0$.

8. i. Show that the volume of the tetrahedron having $\vec{A}+\vec{B},\ \vec{B}+\vec{C},\ \vec{C}+\vec{A}$ as concurrent edges is twice the volume of the tetrahedron having $\vec{A},\vec{B},\vec{C}$ as concurrent edges. **5+5=10**

ii. (ii) If $y_1 = e^{-x}\cos x,\ y_2 = e^{-x}\sin x$ and $\dfrac{d^2y}{dx^2} + \dfrac{2dy}{dx} + 2y = 0$ then

Calculate Wronskian determinant. Verify that the solutions satisfy the differential equation.

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