REV-01
BSP/04/06

2022/12

SET B

B.Sc. PHYSICS
THIRD SEMESTER
MATHEMATICAL PHYSICS-II
BSP – 301

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. Full Marks: 70

(PART-A: Objective)

Time: 30 min. Marks: 20

Choose the correct answer from the following: ***1×20=20***

1. If $1.3.5.......(2n-1)=\frac{2^n}{\sqrt{\pi}}\Gamma(p)$, then p is

a. $n+\frac{2}{3}$ b. $n+\frac{1}{2}$

c. $n+\frac{1}{3}$ d. $\frac{n}{2}-1$

[2]

2. The matrix A is defined as $A=\begin{bmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & 2\end{bmatrix}$. The eigenvalues of A^2 are

a. -1, -9, -4 b. 1, 9, 4

c. -1, -3, 2 d. None of these

3. Rank of the matrix $\begin{bmatrix}1 & 2 & 3\\ 1 & 4 & 2\\ 2 & 6 & 5\end{bmatrix}$ is

a. 2 b. 1

c. 3 d. 0

4. A square matrix A is called an unitary matrix if

a. $A^{\theta}A=I$ b. $\frac{A^{\theta}}{A}=I$

c. $A^{\theta}=I$ d. $A^{\theta}A=0$

5. Find the sum and product of the eigen value of the matrix $\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$

a. 7, 5 b. 9, 5

c. 7, 10 d. 9, 27

USTM/COE/R-01

6. The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigen value is

a. -1
b. 0
c. 2
d. 1

7. x= a is called an ordinary point of the differential equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$, when

a. $P=\infty, Q\neq\infty$
b. $P\neq\infty, Q\neq\infty$
c. $P=\infty, Q=\infty$
d. $P\neq\infty, Q=\infty$

8. $\frac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})}$

a. 2
b. -2
c. $\frac{1}{2}$
d. $-\frac{1}{2}$

9. If A and B are the matrices of same order such that $AB = A$ and $BA = B$, then A and B are

a. Nilpotent
b. Idempotent
c. Singular
d. Hermitian

10. If we interchange the rows and column of a matrix then the new matrix obtained is known as

a. symmetric
b. square
c. transpose
d. triangular

11. The polynomial $2x^2+x+3$ in terms of Legendre's polynomial is

a. $\frac{1}{3}[4P_2-3P_1+11P_0]$
b. $\frac{1}{3}[4P_2+3P_1-11P_0]$
c. $\frac{1}{3}[4P_2+3P_1+11P_0]$
d. $\frac{1}{3}[4P_2-3P_1-11P_0]$

12. $\int_0^\infty e^{-t^2}\,dt=?$

a. $\sqrt{\pi}$
b. $\frac{\sqrt{\pi}}{2}$
c. π
d. 0

13. If x=0 is a regular singular point of the differential equation $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$ and m_1, m_2 are real and different roots then

a. $y=c_1(y)_{m_1}+c_2(y)_{m_2}$

b. $y=c_1(y)_{m_1}+c_2(\frac{dy}{dm})_{m_1}$

c. $y=c_1(y)_{m_1}$

d. $y=c_1(y)_{m_1}+c_2(y)_{m_1}$

14. What is the value of $\beta(z,1)$

a. $\frac{1}{z}$

b. $\frac{1}{z+1}$

c. $\frac{1}{z(z+1)}$

d. $\frac{1}{z-1}$

15. How many constants are require to make a 2nd order partial differential equation

a. 3

b. 2

c. 1

d. 0

16. Complete solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x \partial y}+4\frac{\partial^2 z}{\partial y^2}=0$ is

a. $z=f_1(y+2x)+xf_2(y+2x)$

b. $z=f_1(2y+x)+xf_2(2y+x)$

c. $z=f_1(y+x+2)+xf_2(y+x+2)$

d. $z=f_1(y-2x)+xf_2(y-2x)$

17. The Rodrigue formula for Legendre Polynomial $P_n(x)$ is given by

a. $P_n(x)=\frac{n!}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$

b. $P_n(x)=\frac{n!}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^{n-1}$

c. $P_n(x)=\frac{n!}{2^{n-1}}\frac{d^n}{dx^n}(x^2-1)^n$

d. $P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$

18. A partial differential equation has

a. one independent variable

b. two or more independent variables

c. more than one dependent variable

d. equal number of dependent and independent variables

19. If $P_n(x)$ and $Q_n(x)$ are two independent solution of Legendre equation then the general solution is

a. $y=AP_n(x)+BQ_n(x)$,

b. $y=AP_n^2(x)+BQ_n(x)$

c. $y=AP_n^2(x)+BQ_n^2(x)$

d. None of these

20. The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2} = \sin(xy)$

a. $z = -x^{-2}\sin(xy) + xf(x) + g(x)$

b. $z = -x^{2}\sin(xy) - yf(x) + g(x)$

c. $z = -y^{2}\sin(xy) + yf(x) + g(x)$

d. $z = -x^{2}\sin(xy) + yf(x) + g(x)$

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PART-B : Descriptive

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. i. Form the partial differential equation from $z=(x+a)(y+b)$ 1+5+4 =10

   ii. By method of separation of variables solve the partial differential equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.

   iii. Solve $\frac{\partial^3 z}{\partial x^2 \partial y}=\cos(2x+3y)$

2. i. Find the deflection $u(x,y,t)$ of the square membrane with $a=b=c=1$, if the initial velocity is zero and the initial deflection $f(x,y)=A\sin\pi x\sin 2\pi y$. 10

3. i. Prove that $\int_0^1 \frac{x^2}{\sqrt{(1-x^4)}}dx \times \int_0^1 \frac{dx}{\sqrt{(1+x^4)}}=\frac{\pi}{4\sqrt{2}}$ 5+3+2 =10

   ii. Prove that $\int_0^1 x^m(\log x)^n dx=\frac{(-1)^n}{(m+1)^{n+1}}\overline{|n+1}$

   iii. Evaluate $\overline{|-\frac{1}{2}}$

4. i. Using Frobenius methods solve $x(x-1)y''+(3x-1)y'+y=0$. 7+3=10

   ii. Express the polynomial $f(x)=4x^3-2x^2-3x+8$ in terms of Legendre Polynomials.

5. 1+3+3+3 =10

(i) Define orthogonal matrix.

(ii) Verify that $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.

(iii) Determine the values of α, β, γ when $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal.

(iv) Prove that $(AB)^n = A^n.B^n$, if $A.B = B.A$

6. (i) Show that any square matrix can be expressed as the sum of two matrices, one symmetric and other anti-symmetric. 2+4+4 =10

(ii) The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. Find the eigen values of $3A^3 + 5A^2 - 6A + 2I$ where I is the unit matrix of order 3.

(iii) Prove that if the product of two matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is zero then θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

7. i. Evaluate Beta function in terms of gamma function. **4+4+2 =10**

ii. Prove that $1.3.5....(2n-1)=\dfrac{2^n\Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}}$

Show that $\beta(l,m)=\beta(m,l)$.

8. (i) Find regular singular points of the differential equation $x(x-2)^2y''+2(x-2)y'+(x+3)y=0$. **3+3+4 =10**

(ii) Draw and explain the graph for Legendre's polynomial P_0, P_1, P_2, P_3, P_4.

(iii) Solve $\dfrac{\partial^3 z}{\partial x^2 \partial y}=\cos(2x+3y)$

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