REV-01
BSP/08/13

2022/12

B.SC. PHYSICS
FIFTH SEMESTER
QUANTUM MECHANICS & APPLICATIONS
BSP – 501

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. For the stationary states, one of the following is true. Which one?
   ($|\Psi(x,t)|^2$ is the probability density and $\langle Q(x,p)\rangle$ is the expectation value of any dynamical variable)
   a. $|\Psi(x,t)|^2$ and $\langle Q(x,p)\rangle$ both are constant in time.
   b. $|\Psi(x,t)|^2$ is constant in time while $\langle Q(x,p)\rangle$ is not.
   c. $|\Psi(x,t)|^2$ is constant in time while $\langle Q(x,p)\rangle$ is not.
   d. $|\Psi(x,t)|^2$ and $\langle Q(x,p)\rangle$ both are not constant in time.

2. The commutator between two operators A and B is defined as
   a. $[A,B] = AB - BA$
   b. $[A,B] = AB + BA$
   c. $[A,B] = A + B$
   d. $[A,B] = A - B$

3. A wave function has the form, $\psi(x,t) = Ae^{-it}$ (A is real, independent of both x and t). We can conclude that
   a. The probability density is zero.
   b. The probability density oscillates with time.
   c. The probability density is a constant over time.
   d. The probability density decays with time.

4. The energy difference between adjacent simple harmonic oscillator (1D) energy levels is
   a. $\frac{1}{2}\hbar\omega$
   b. $\hbar\omega$
   c. $\frac{3}{2}\hbar\omega$
   d. $2\hbar\omega$

5. The operator that represents the kinetic energy in quantum mechanics is
   a. $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
   b. $\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
   c. $-\frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}$
   d. $\frac{2m}{\hbar^2}\frac{\partial^2}{\partial x^2}$

6. a_- is the lowering operator and ψ_0 is the ground state for the simple harmonic oscillator. The correct choice is
   a. $a_-\psi_0 = A\psi_{-1}$
   b. $a_-\psi_0 = A\psi_1$
   c. $a_-\psi_0 = A\psi_0$
   d. $a_-\psi_0 = 0$

7. σ_x and σ_p are the standard deviations in x and p, respectively. Heisenberg's uncertainty principle states that

a. $\sigma_x\sigma_p = 0$

b. $\sigma_x\sigma_p \leq 0$

c. $\sigma_x\sigma_p \geq \frac{\hbar}{2}$

d. $\sigma_x\sigma_p \leq \frac{\hbar}{2}$

8. The infinite square well potential has the form $V(x) = \begin{cases} 0, \text{if} -a \leq x \leq a \\ \infty, \text{ otherwise} \end{cases}$

The probability of finding the particle within the range $-5a \leq x \leq -2a$ is

a. 1

b. 0

c. 0.5

d. Between 0.1 to 0.2

9. For a free particle in quantum mechanics, the group velocity v_{group} and phase velocity v_{phase} are related as

a. $v_{group} = 2v_{phase}$

b. $v_{group} = \frac{1}{2}v_{phase}$

c. $v_{group} = v_{phase}$

d. $v_{group} = 3v_{phase}$

10. Choose the right commutation relation between x and p.

a. $[x, p] = -\hbar$

b. $[x, p] = \hbar$

c. $[x, p] = i\hbar$

d. $[x, p] = -i\hbar$

11. The Gyromagnetic ratio of the electron spin is given by

a. $\frac{\vec{\mu}_l}{\vec{L}} = \sqrt{l(l+1)}\hbar$

b. $\frac{\vec{\mu}_l}{\vec{S}} = -\frac{e}{m}$

c. $\frac{\vec{\mu}_l}{\vec{S}} = -\frac{e}{2m}$

d. $\frac{\vec{\mu}_l}{\vec{L}} = -\frac{eV}{2\pi r}$

12. Orbital angular momentum is classically expressed as

a. $\vec{L} = \frac{\vec{r} \times \vec{v}}{m}$

b. $\vec{L} = \frac{m\vec{r}}{\vec{v}}$

c. $\vec{L} = m(\vec{r} \times \vec{v})$

d. $\vec{L} = \frac{\vec{r}}{m\vec{v}}$

13. The magnitude of the spin magnetic dipole moment is given by

a. $\mu_s = \frac{1}{2}\frac{e\hbar}{m}$

b. $\mu_s = \frac{e\hbar}{4\pi m}$

c. $\mu_s = \frac{\sqrt{3}}{2}\frac{e\hbar}{m}$

d. $\mu_s = 0.866\hbar$

14. For L-shell (n=2), the number of possible sets of quantum numbers is

a. 2

b. 4

c. 6

d. 8

15. Quantization of orbital angular momentum is represented as

a. $L = n\pi\hbar$

b. $L = mvr/\hbar$

c. $L = n\hbar$

d. $L = n\pi/\hbar$

16. The calculated value of the Rydberg constant is

a. $R = 1.097 \times 10^7/m$

b. $R = 1.67 \times 10^{-27}/m$

c. $R = 9.11 \times 10^{-31}/m$

d. $R = 3 \times 10^8/m$

17. The separation between the Sodium D lines in terms of wavelength is

a. $\Delta\lambda = 0.6\ nm$
b. $\Delta\lambda = 6\ nm$
c. $\Delta\lambda = 60\ nm$
d. $\Delta\lambda = 600\ nm$

18. The spectral line series of H-atom which fall in visible range of wavelength is

a. Pfund
b. Bracket
c. Lyman
d. Balmer

19. The total energy E, contain in the radial wave equation of the H-atom, indicates the electron is bound to the proton, if

a. E is positive
b. E is negative
c. E is zero
d. E is imaginary

20. Energy in the first Bohr's orbit of H-atom is

a. 7.1 eV
b. 1.13 eV
c. - 4.01 eV
d. -13.6 eV

-- --- --

Descriptive

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. At time $t = 0$, a particle is represented by the wave function **4+2+1+3=10**

$$\Psi(x,0) = \begin{cases} \frac{Ax}{a} & \text{if } 0 \le x \le a, \\ \frac{A(b-x)}{b-a} & \text{if } a \le x \le b, \\ 0 & \text{otherwise,} \end{cases}$$

Where $A, a,$ and b are constants.

(a) Normalize Ψ (that is, find A in terms of a and b).

(b) Sketch $\Psi(x,0)$ as a function of x.

(c) Where is the particle most likely to be found, at $t = 0$?

(d) What is the probability of finding the particle to the left of a? Check your result in the limiting cases $b = a$ and $b = 2a$.

2. a. Draw the first three stationary states of the infinite square well (bounded between 0 to a). Identify the number of nodes in each of them. **4+3+3=10**

b. A particle in the infinite square well has its initial wave function an even mixture of the first two stationary states: $\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]$.

(i) Normalize $\Psi(x,0)$. [No explicit integration is allowed]

(ii) Find $\Psi(x,t)$.

3. The ladder operators are defined as $a_\pm \equiv \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$. **4+6=10**

a. Find out the commutator between a_- and a_+, that is $[a_-, a_+]$ =?

b. Find the expectation value of the potential energy in the nth stationary state of the harmonic oscillator.

4. A free particle, which is initially localized in the range $-a < x < a$, is released at time $t = 0$: **2+3+2+3=10**

$$\Psi(x,0) = \begin{cases} A & \text{if } -a < x < a \\ 0 & \text{otherwise,} \end{cases}$$

Where A and a are positive real constants.

a. Normalize $\Psi(x,0)$.

b. Find $\phi(k)$ using the relation $\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \Psi(x,0)e^{-ikx}dx$.

c. Construct $\Psi(x,t)$ following the relation

$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{-i\left(kx - \frac{\hbar k^2}{2m} t\right)} dk$. [Please note that you cannot solve it analytically. This could be done numerically. So just concentrate on the construction of the wave function.]

d. Discuss the limiting cases for $\Psi(x,0)$ and $\phi(k)$ (a very large, and a very small).

5. State and explain Pauli's exclusion principle. Discuss the distribution of quantum numbers in L-shell, following Pauli's exclusion principle **4+6=10**

6. a. What is Bohr's magnetron? Calculate the Bohr's magnetron for the ground state electron in H-atom (Given $\hbar = 1.05 \times 10^{-34}$ Js). **1+4+2+3=10**

   b. Deduce the magnitudes of the orbital dipole magnetic moment $\vec{\mu}_l$ of the electron in H-atom in p- and d-states. (Assume the spin of electron is zero)

7. Assuming the postulates of Bohr's theory of the Hydrogen atom, derive the expressions for (i) radius of the Bohr's orbit, and (ii) energy of the electron in n-th orbit. **4+6=10**

8. a. State and explain Pauli's exclusion principle. **3+3+4=10**

   b. Discuss the distribution of quantum numbers in K- and L-shells, following Pauli's exclusion principle.

= = *** = =