M.Sc. PHYSICS
SECOND SEMESTER
ELECTROMAGNETICS & PLASMA PHYSICS
MSP – 201

(Use Separate Answer Scripts for Objective & Descriptive)

Duration: 3 hrs. Full Marks: 70

(PART-A: Objective)

Time: 20 min. Marks: 20

Choose the correct answer from the following: ***1X20=20***

1. The potential for a dipole goes as
   a. $V \sim \frac{1}{r}$
   b. $V \sim \frac{1}{r^2}$
   c. $V \sim \frac{1}{r^3}$
   d. $V \sim \frac{1}{r^4}$

2. In 3D, the Laplace's equation reads as (constants have their usual meanings)
   a. $\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} = 0$
   b. $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$
   c. $\frac{\partial^2 V}{\partial x \partial y} + \frac{\partial^2 V}{\partial y \partial z} + \frac{\partial^2 V}{\partial z \partial x} = 0$
   d. $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\rho/\epsilon_0$

3. If V has azimuthal symmetry, it depends on
   a. r only
   b. θ only
   c. ϕ only
   d. r and θ only

4. . In case of a 2D system, to solve the Laplace's equation, in general, the required number of boundary conditions is
   a. 1
   b. 2
   c. 3
   d. 4

5. $f(z,t)$ represents a wave of fixed shape traveling in the z direction at speed v. One of the following forms does not represent a wave. Which one?
   a. $Ae^{-b(z-vt)^2}$
   b. $Ae^{b(bz^2+vt)}$
   c. $\frac{A}{b(z-vt)^2}$
   d. $A\sin[b(z-vt)]$

6. $\tilde{E}(z,t) = \tilde{E}_0 e^{i(kz-\omega t)}\hat{x}$, represents the electric field of an electromagnetic (EM) wave. The polarization of the field is along the
   a. x direction
   b. y direction
   c. z direction
   d. Parallel to the $x-z$ plane

7. The allowed gauge transformations for the scalar and vector potentials are
   a. $A' = A + \nabla\lambda, V' = V - \frac{\partial \lambda}{\partial t}$
   b. $A' = A - \nabla\lambda, V' = V + \frac{\partial \lambda}{\partial t}$
   c. $A' = A + \nabla\lambda, V' = V + \frac{\partial \lambda}{\partial t}$
   d. $A' = A - \nabla\lambda, V' = V - \frac{\partial \lambda}{\partial t}$

8. In the Lorentz gauge, we pick

a. $\nabla \cdot A = 0$

b. $\nabla \cdot A = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

c. $\nabla \cdot A = \mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

d. $\nabla \times A = 0$

9. Fresnel's equation states that the transmitted wave is

a. in phase with the incident one

b. 180° out of phase with the incident one

c. 90° out of phase with the incident one

d. None of these

10. For an electromagnetic (EM) plane wave $\hat{n}$ and $\hat{k}$ are the polarization vector and the direction of wave vector, respectively. They are related as

a. $\hat{n} \times \hat{k} = 0$

b. $\hat{n} \cdot \hat{k} = 0$

c. $\hat{n} \cdot \hat{k} \neq 0$

d. $\hat{n} \cdot \hat{k} \neq 0$

11. If λ_D is the Debye length and L is the dimension of plasma, then which of the following represents the criterion of an ionized gas to behave as plasma?

a. $\lambda_D << L$

b. $\lambda_D >> L$

c. $\lambda_D = L$

d. $\lambda_D = \sqrt{L}$

12. Drift of a gyrating particle in crossed gravitational and magnetic fields depends on the ratio (the symbols have their usual meanings)

a. $\frac{mv_\parallel^2}{q}$

b. $\frac{mv_\perp^2}{q}$

c. $\frac{m}{q}$

d. $\frac{v_\parallel^2}{2m}$

13. Suppose we have a vacuum chamber with electric field $E = 10\ \text{Vm}^{-1}$ perpendicular to magnetic field $B = 1$ mT. The $E \times B$ drift speed for an electron inside the chamber is

a. 103 m/s

b. 10^4 m/s

c. 1 m/s

d. 10^{-3} m/s

14. The gradient drift for a charge particle of mass m and charge q moving in a system with non-uniform magnetic field $\vec{B}$ is (the symbols have their usual meanings)

a. $\vec{v}_{\nabla B} = \pm \frac{mv_\perp^2}{2q} \frac{\vec{B} \times \nabla \vec{B}}{B^3}$

b. $\vec{v}_{\nabla B} = \frac{1}{2} mv_\perp^2 \frac{\vec{B} \times \nabla \vec{B}}{B^3}$

c. $\vec{v}_{\nabla B} = \pm \frac{mv_\parallel^2}{2qB} \frac{\vec{B} \times \nabla \vec{B}}{B^2}$

d. $\vec{v}_{\nabla B} = \pm \frac{mv_\perp}{2qB} \frac{\vec{B} \times \nabla \vec{B}}{B^2}$

15. Plasma oscillations is

a. directly proportional to the square root of number density of electrons

b. inversely proportional to the square root of mass of electron

c. directly proportional to the charge of electron

d. directly proportional to the thermal velocity of electrons.

16. For an electron plasma wave, if v_{th} is thermal velocity of electrons, the phase velocity v_{ph} is always greater than or equal to

a. $\sqrt{3/2}\, v_{th}$

b. $\sqrt{1/3}\, v_{th}$

c. $2v_{th}$

d. v_{th}^2

17. The Alfven waves in a compressible, conducting fluid of density ρ_m immersed in a magnetic field of strength B_0 is given by (symbols have their usual meanings)

a. $$v_A = \sqrt{\frac{B_0}{\mu_0 \rho_m}}$$

b. $$v_A = \frac{B_0}{\sqrt{\mu_0 \rho_m}}$$

c. $$v_A = \frac{\rho_m}{\sqrt{\mu_0 B_0}}$$

d. $$v_A = \frac{1}{\mu_0}\sqrt{\frac{B_0^2}{\rho_m^2}}$$

18. Rayleigh scattering is strongly dependent upon the

a. size of the particle

b. mass of the particle

c. charge of the particle

d. spin of the particle

19. The four momentum of a particle at rest having mass m is

a. $$p^\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

b. $$p^\mu = \begin{pmatrix} mc^2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

c. $$p^\mu = \begin{pmatrix} mc \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

d. $$p^\mu = \begin{pmatrix} mc^2 \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}$$

20. Which of the following is a correct description of 4-potential [symbols have their usual meaning]

a. $$A^\mu = \begin{pmatrix} \phi \\ \vec{A} \end{pmatrix}$$

b. $$A^\mu = \begin{pmatrix} \phi/c \\ \vec{A} \end{pmatrix}$$

c. $$A^\mu = \begin{pmatrix} c\phi \\ \vec{A} \end{pmatrix}$$

d. $$A^\mu = \begin{pmatrix} \phi \\ \vec{A}/c \end{pmatrix}$$

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PART-B : Descriptive

Time : 2 hrs. 40 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Find $P_3(x)$. The Legendre polynomial $P_l(x)$ is defined by the Rodrigues formula $P_l(x) = \frac{1}{2^l}\left(\frac{d}{dx}\right)^l (x^2-1)^l$. 5+5=10

   b. Derive an expression for $\vec{E} \times \vec{B}$ drift velocity for the motion of non-relativistic charged particles in presence of crossed electric ($\vec{E}$) and magnetic ($\vec{B}$) fields which are constant in time and uniform in space.

2. a. Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the $x - z$ direction. Give the explicit Cartesian components of $\vec{k}$ and $\hat{n}$. 5+4+1 =10

   b. Derive an expression for the Debye shielding length of a typical plasma system.

   c. What do you mean by pinch effect in plasma?

3. a. A sphere of radius R, centered at the origin, carries charge density 2+2+6 =10

$$\rho(r,\theta) = k\frac{R}{r^2}(R-2r)\sin\theta,$$

Where k is a constant, and r, θ are the usual spherical coordinates. Find the approximate potential for points on the z axis, far from the sphere. [Help: $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\int (r')^n P_n(\cos\theta')\rho\left(\vec{r'}\right)d\tau'$.]

4. The potentials for a particular charge and current distributions are 4+4+2 =10

$$V = 0, \qquad A = \begin{cases} \frac{\mu_0 k}{4c}(ct-|x|)^2\hat{z} & \text{for } |x| < ct \\ 0 & \text{for } |x| > ct \end{cases}$$

Where k is a constant and $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$.

   a. Find the electric and magnetic fields. $\left[E = \nabla V - \frac{\partial A}{\partial t}\right]$

   b. Show that the computed electric and magnetic fields satisfy the Maxwell's equations.

   c. What are the charge and current densities?

5. a. The potential $V_0(\theta) = k\sin^2\left(\frac{\theta}{2}\right)$ is specified on the surface of a hollow sphere, of radius R. Find the potential inside the sphere. [Hint: $V(r,\theta) = \sum_{l=0}^{n}\left(A_l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$, think about the origin, which factor is going to blow up. Try to write down the potential in terms of the Legendre polynomials and then equate.] **5+5=10**

b. The potential $V_0(\theta)$ is again specified on the surface of a sphere of radius R. Find the potential outside the sphere, assuming that no charge is there.

6. a. In a magnetic mirror, the trajectory of a charged particle makes an angle θ_0 with the magnetic field line. Show that the mirror ratio R_m is given by **8+2=10**

$\frac{1}{R_m} = \frac{B_0}{B_m} = \sin^2\theta_m$.

Here B_0 is the strength of the magnetic field at the centre and B_m is the strength of the magnetic field at the throat of the magnetic mirror system.

b. A plasma with an isotropic velocity distribution is placed in a magnetic mirror trap with mirror ratio $R_m = 2$. Find the value of pitch angle θ_m.

7. Show that the dispersion relation for electron plasma wave is given by **10**

$\omega^2 = \omega_p^2 + \frac{3}{2}K_B^2 v_{th}^2$.

Hence demonstrate the graphical representation of the variation of plasma frequency ω with wave vector k. (symbols have their usual meanings)

8. Starting from the electromagnetic field tensor $F^{\mu\nu}$, derive Maxwell's first and third equations. **10**

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