

**B.Sc. MATHEMATICS  
FOURTH SEMESTER  
CALCULUS  
BSM-741**

Duration : 3 hrs.

Full Marks: 70

[ PART-A: Objective ]

Time : 20 min.

Marks : 20

Choose the correct answer from the following:

1X20=20

1. If  $f(x) = x^2 \sin \frac{1}{x}, x \neq 0$ , then the value of the function  $f(x)$  at  $x = 0$ , so that the function is continuous at  $x = 0$  is
  - a. 0
  - b. -1
  - c. 1
  - d. None of the above
2. The set of points where the function  $f$  given by  $f(x) = |2x - 1| \sin x$  is differentiable is
  - a.  $\mathbb{R}$
  - b.  $\mathbb{R} - \left\{ \frac{1}{2} \right\}$
  - c.  $(0, \infty)$
  - d. None
3. The function  $f(x) = \cot x$  is discontinuous on the set
  - a.  $\{x = n\pi, n \in \mathbb{Z}\}$
  - b.  $\{x = 2n\pi, n \in \mathbb{Z}\}$
  - c.  $\left\{ x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \right\}$
  - d. None
4. The function  $f(x) = \frac{4-x^2}{4x-x^3}$  is
  - a. Discontinuous at only one point at  $x = 0$
  - b. Discontinuous at exactly two points
  - c. Discontinuous at exactly three points
  - d. None



11. If  $y = \cos^2 x$  then nth derivative is

a.  $2^{n-1} \cos\left(\frac{n\pi}{2} + x\right)$

b.  $2^{n-1} \cos\left(\frac{n\pi}{2} + 2x\right)$

c.  $2^{n-1} \cos(2n\pi + 2x)$

d. None

12. The function  $f(x) = |x - 3|$  is continuous at

a.  $x = 3$

b.  $x = -3$

c. Both a and b

d. None

13. The integrals of  $\int \frac{\cos(\log x)}{x} dx$  is

a.  $\log(\sin x)$

b.  $\sin\left(\log \frac{1}{x}\right)$

c.  $\sin(\log x)$

d. None

14.  $\int \frac{dx}{e^x + e^{-x}}$  is equals to

a.  $\log(e^x)$

b.  $\tan^{-1}(e^x)$

c.  $e^{\tan^{-1} x}$

d. None

15. The value of  $\int \frac{dx}{x^2 + 1}$  is

a. 1

b. 0

c.  $\tan^{-1} x$

d. None

16. Rate of change of area of circle with respect to radius is

a. Circumference

b.  $2\pi r$

c. Both a and b

d. None

17. Which one is correct?

a.  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

c.  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

b.  $\lim_{x \rightarrow 0} \frac{\cos x}{x} = 1$

d. Both a and c

18. The interval of  $(x-a)(x-b) < 0$  is

a.  $(a, b)$

c.  $[a, b)$

b.  $[a, b]$

d. None

19. The value of  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$  is

a.  $\frac{3}{2}$

c. 0

b.  $\frac{2}{3}$

d. None

20. The value of  $\lim_{x \rightarrow \infty} \frac{2}{x}$  is

a. 0

c. 2

b.  $\infty$

d. None

**( PART-B : Descriptive )**

Time: 2 HRS 40 MINS

Marks : 50

[ Answer question no.(1) & any four (4) from the rest ]

1. Prove that if a function is differentiable at a point, it is necessarily continuous at that point. But the converse is not true. 10

2. Discuss the continuity and differentiability of 10

$$f(x) = \begin{cases} 1-x, & x < 1 \\ (1-x)(2-x), & 1 \leq x \leq 2 \\ 3-x, & x > 2 \end{cases}$$

3. a. Find the positive root of  $x^4 - x = 10$  correct to three decimal places using Newton's method. 5+5=10

b. Evaluate  $\sqrt{5}$  to four decimal places by Newton's iteration method.

4. Integrate 5+5=10

a.  $\int \sin^{-1} \sqrt{\frac{x}{x+a}} dx$

b.  $\int \frac{dx}{x^3 - x^2 - x + 1}$

5. Find  $\frac{dy}{dx}$  5+5=10

a.  $\sin^{-1} \left( 2x\sqrt{1-x^2} \right), \frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

b.  $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

6. a. Find the rate of change of volume of a sphere with respect to its surface area when the radius is 2 cm. 5+5=10

b. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/sec. How fast its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

7. If  $y = \cos(m \sin^{-1} x)$  then prove that 10

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

8. If  $I_n = \int_0^{\pi/2} \theta \sin^n \theta d\theta$  and  $n > 1$  prove that  $I_n = \frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$  10

also evaluate  $I_5$ .

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