B.Sc. PHYSICS
SIXTH SEMESTER
MATHEMATICAL PHYSICS-III
BSP - 603A

(Use Separate Answer Scripts for Objective & Descriptive)

Duration: 3 hrs.
Full Marks: 70

[PART-A: Objective]

Time: 20 min.
Marks: 20

Choose the correct answer from the following:
1X20=20

1. The magnitude of the complex number $(-4 + 3i)$ is
   - a. -4
   - b. 3
   - c. 5
   - d. -1

2. The value of i^i is
   - a. $e^{\frac{\pi}{2}}$
   - b. $e^{-\frac{\pi}{2}}$
   - c. i
   - d. 1

3. The argument of the sum of the complex numbers $(-5 + 2i)$ and $(4 - 3i)$ is
   - a. $\frac{\pi}{2}$
   - b. $\frac{\pi}{3}$
   - c. $\frac{\pi}{4}$
   - d. $\frac{\pi}{6}$

4. The inverse of the complex number $(-8 + 6i)$ is
   - a. $\frac{1}{5}(-4 - 3i)$
   - b. $\frac{1}{5}(-4 + 3i)$
   - c. $\frac{1}{5}(4 - 3i)$
   - d. $\frac{1}{5}(4 + 3i)$

5. The Laplace transform $F(x)=x$ is
   - a. s^{-1}
   - b. s^{-2}
   - c. s^1
   - d. s^2

6. If $L[F(x)]=f(s)$ then $L[\sin 2x]$ is
   - a. $\frac{2}{s^2 + 4}$
   - b. $\frac{1}{s^2 + 4}$
   - c. $\frac{2}{s^2 + 2}$
   - d. $\frac{1}{s^2 + 2}$

7. If $L[F(x)]=f(s)$ then $L[x^n F(x)]$ is
   - a. $(-1)^n f^{n+1}(s)$
   - b. $(-1)^n f^{n-1}(s)$
   - c. $(-1)^n f^n(s)$
   - d. $(-1)^n f^{-n+1}(s)$

8. $L^{-1}(1/s)$ is
   - a. x^0
   - b. x^{-1}
   - c. x^2
   - d. x^1

[1]

9. The value of $\oint \frac{e^z}{z+1} dz$ for a circle of $|z|=1/2$ is
   a. $2\pi i$ b. πi
   c. 0   d. 1

10. The value of $\oint \frac{dz}{z-a}$ for a circle of $|z-a|=r$ is
   a. 0   b. $2\pi i$
   c. πi d. 2π

11. A "periodic function" is given by a function which
   a. has a period   b. has a period
   c. satisfies   d. satisfies

12. What are the conditions called which are required for a signal to fulfill to be represented as Fourier series?
   a. Dirichlet's conditions   b. Gibbs phenomenon
   c. Fourier conditions   d. Fourier phenomenon

13. For the given periodic function for the coefficient is
   a. 0   b. -6.8968
   c. 6.8968   d. 0.7468

14. A function f(x) is called skew symmetric function if
   a. $f(-x) = -f(x)$ b. $f(-x) = f(x)$
   c. $f(-x) = -f(-x)$ d. $f(-x) = 0$

15. If the Fourier series of f(x) has only cosine terms then f(x) must be
   a. Odd function   b. Even function
   c. Fundamental harmonic   d. Second harmonic

16. In the following function f(x) is known as

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-isx} F(s)ds$$

   a. Fourier transform of f(s)   b. Fourier transform of f(x)
   c. Fourier transform of F(x)   d. Inverse Fourier transform of F(s)

17. $\int_{0}^{\infty} \frac{\sin ax}{x} dx = ?$

   a. $\frac{\sqrt{\pi}}{2}$ b. 0

   c. $\sqrt{\frac{\pi}{2}}$ d. $\frac{\pi}{2}$

18.

Fourier cosine transform of $\dfrac{1}{\sqrt{x}}$

a. $\sqrt{\dfrac{\pi}{2s}}$

b. $\sqrt{\dfrac{\pi}{s}}$

c. $\sqrt{\dfrac{2\pi}{s}}$

d. $\dfrac{1}{\sqrt{s}}$

19. Fill in the blank. The property is known as---------, when F(s) is the complex Fourier transform of f(x) then $F\{f(x-a)\} = e^{isa}F(s)$

A. Modulation theorem

b. Change of scale property

C. Linear property

d. Shifting property

20. Fourier transform of f(t)=-----------x Laplace transform of g(t). Fill in the blank

a. $\dfrac{1}{\sqrt{2\pi}}$

b. $\dfrac{1}{\sqrt{2\pi}}$

c. $\dfrac{1}{\sqrt{\pi}}$

d. None of these

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Time : 2 hrs. 40 min.   Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. Find the solution of the differential equation $\frac{d^2 y}{dx^2} + \frac{dy}{dx} = b\,x$ using Laplace transform with the boundary $y(0)= 1$ and $y'(0) =0$.   **7+3=10**

    b. Find Laplace transform of $x + x^2 + x^3$

2. a. Obtain a Fourier expression for $f(x) = x^3 , -\pi < x < \pi$.   **4+4+2 =10**

    b. Find the Fourier series representing $f(x) = x , 0 < x < 2\pi$.

    c. Represent the following function by a Fourier sine series: $f(t) = \begin{cases} t, \\ \dfrac{\pi}{2}, \end{cases}$

    $$0 < t \le \frac{\pi}{2}$$

    $$\frac{\pi}{2} < t \le \pi$$

3. a. If n is a positive integer, prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$ **3+3+4 =10**

    b. If ω is a cube root of unity, find $(1 - \omega)^6$?

    c. Express the complex number $\frac{1+3\,i}{1-2\,i}$ in polar form.

4. A machine completes its cycle of operations every time as certain pulley completes a revolution. The displacement $f(x)$ of a point on a certain portion of the machine is given in the table below for twelve positions of the pulley, x being the angle in degree turned through by the pulley. Find a Fourier series to represent $f(x)$ for all values of x.   **10**

x	30⁰	60⁰	90⁰	120⁰	150⁰	180⁰	210⁰	240⁰	270⁰	300⁰	330⁰	360⁰
f(x)	7.976	8.026	7.204	5.674	3.674	1.764	0.552	0.262	0.904	2.492	4.736	6.824

5. a. Using Cauchy's integral formula evaluate $\oint \frac{z}{z^2-3z+2} \, dz$ for a circle $|z-2|=1/2$.   4+3+3 =10

    b. Find the inverse transform of $\frac{s+2}{(s+2)^2+9}$

    c. Find the Laplace transform L [F(t)] if

$$F(t)=\begin{cases} \sin(t-\frac{\pi}{3}), & t > \frac{\pi}{3} \\ 0, & t < \frac{\pi}{3} \end{cases}$$

6. a. Discuss the linear property of Fourier transform.   5+5=10

    b. Prove that the Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transform.

7. a. Find the value of x and y if $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$.   4+3+3 =10

    b. If $(x+iy)^{1/3} = a+ib$ then show that $4(a^2-b^2) = \frac{x}{a}+\frac{y}{b}$.

    c. Show that $L\left[\frac{1}{\sqrt{\pi x}}\right] = s^{-1/2}$.

8. An infinitely long string having one end at x=0 is initially at rest along x-axis. The end x=0 is given a transverse displacement f(t), when t > 0. Find the displacement of any point of the string at any time.   10

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